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Study of Effects of Elliptic Plates and Comparison with other Lubrication Models using Incompressible Second-Order Fluid

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Abstract

A near investigation of non-Newtonian consequences for different parallel plates contrasted with endlessly/infinite long rectangular, circular, elliptical, equilateral triangular plates lubricated by incompressible second order fluids on the performance characteristics by squeeze film bearing are displayed. This model is the least difficult sort of model which accurately represent where the oil is subjected to quickly changing conditions along its streaming way, such time reliance is the second order fluid. The fundamental lubricant equations are found from the original conditions. It is discovered that as in Newtonian liquids, squeeze is continuously quicker as we go from rectangular plates to equilateral triangular plates and the span required declines as the second-order parameter increments.

Keywords: Non-Newtonian Fluid, Elliptic Plates, Rectangular Plates, Second order fluid, Incompressible Fluid

1. Introduction

The squeeze film phenomenon emerges when the two greasing up surfaces move towards each other the typical way and produces a positive pressure, and henceforth supports a load. This is because of the way that a thick lubricant exhibit between the two surfaces can't be instantaneously crushed out when the two surfaces move towards each other and this activity gives a padding impact in orientation. The squeeze film lubrication between two limitlessly long parallel plates is contemplated by Cameron (1981). The established continuum mechanism of fluids ignores the span of liquid particles in the stream of liquids, and henceforth several micro-continuum theories have been proposed to consider the inherent movement of material constituents (Ariman et.al, 1973, 1974). In 2014, Siddangouda Apparao etal [1] has studied the Non-Newtonian effects of second-order fluids on the hydrodynamic lubrication of inclined slider bearings.

Whenever an analysis of squeeze films or porous bearing is made it has been standard to accept that for the stream/flow field outside the permeable/porous medium, the tangential velocity components are zero at the surface of the permeable material.

A non-newtonian liquid which doesn't take after Newton's law of thickness. For the most part, the thickness of non-newtonian liquids depends on shear rate or shear rate history. Numerous salt solution and liquid polymers are categorized into non-newtonian liquids, and numerous regularly discovered substances, for example, ketchup, custard, starch syrup and so forth. There are a few properties demonstrating non-newtonian conduct, for example, shear thickening (dilatants), and shear diminishing (pseudoplastic). Different specialists have contemplated non-newtonian conduct utilizing diverse bearing frameworks and attributes, for example, H.K.Van Poollen, J.R.Jagan[2] has considered steady and non-enduring conduct/ unsteady of stream of non-newtonian liquids utilizing porosity factor.

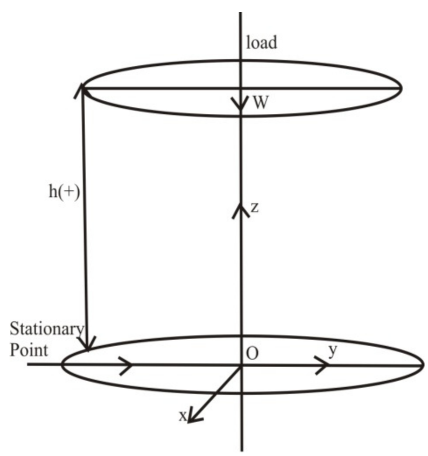
Q.O.Ajadi etal [3] has studied the non-newtonian fluid flow under slip boundary layer over a bearing Plate-flat plate with relevant boundary conditions. Then again case considered by Naveen Singhal [4], the examination investigation of friction pressure correlation for Newtonian and non-newtonian liquid in concentric annuli.

A second order fluid is where the pressure tensor is the total of every one of the tensors that can be framed from the velocity field with up to two subordinate. Numerous analysts have drawn consideration towards second order fluid; in 1994 A.M.Siddiqui [5] has examined the body functional flow of a second order fluid in tubes, though in Oct 2003, M.Emim Erdogan [6] contemplated the changeable motion of a second order fluid over a plane wall.

2. Experimental Methods

The two parallel surfaces moving relative with each other are called as the squeezing film between two parallel plates is shown in Figure (1):

Figure 1: Squeeze Film between Parallel Plates



The lower plate is motionless whereas the upper plate moves slowly towards the lower plate.

The governing constitutive equations are taken:

$$\tau_{ij} = -p\delta_{ij} + \varphi_1 A_{ij} + \varphi_2 B_{ij} + \varphi_3 A_{ik} A_{jk} \quad (1.1)$$

Where

$$A_{ij} = V_{i,j} + V_{j,i} \quad (1.2)$$

$$B_{ij} = a_{i,j} + a_{j,i} + 2V_{k,i} V_{k,j} \quad (1.3)$$

Where τ_{ij} is the stress tensor, δ_{ij} is the Kroneker delta, v_i is the velocity vector, a_i the acceleration vector, p the pressure and comma (,) denotes the partial differentiation.

Under the assumption

a) Fluid between the plates is incompressible.
b) Fluid is second order fluid.

The field equation of the fluid are

$$V_{i,i} = 0 \quad (1.4)$$

$$\rho a_i = \tau_{ij,j} \quad (1.5)$$

Where ρ is the density of the fluid.

According to the assumptions of hydrodynamic lubrication, the equation (1.4) and (1.5) takes the form,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1.6}$$

$$\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial z^2}+\gamma\frac{\partial^3 u}{\partial z^2\partial t} \tag{1.7}$$

$$\frac{\partial p}{\partial y}=\mu\frac{\partial^2 v}{\partial z^2}+\gamma\frac{\partial^3 v}{\partial z^2\partial t} \tag{1.8}$$

$$\frac{\partial p}{\partial z}=0 \tag{1.9}$$

Where (u, v, w) is the velocity vector, μ denotes the coefficient of viscosity and γ, represents the parameter peculiar to the second order fluid.

3. Analysis

The Boundary Condition assumed to be

$$u=0, v=0, w=0 \text{ at } z=0 \tag{2}$$

$$u=v=0,\ w=\dot{h}=\tfrac{dh}{dt} \text{ at } z=h \tag{2.1}$$

where h (t) is the film thickness.

As we know p is not influenced by z, we can solve equation (1.7) and (1.8) using the boundary condition (2) and (2.1) and obtain,

$$\frac{1}{2}\frac{\partial p}{\partial x}\left(z^2-zh\right)=\mu u+\gamma\frac{\partial u}{\partial t} \tag{2.2}$$

$$\mu v+\gamma\frac{\partial v}{\partial t}=\frac{1}{2}\frac{\partial p}{\partial y}\left(z^2-zh\right) \tag{2.3}$$

Integrating the equation of continuity (1.6) across the fluid film yields using (2), we get,

$$\int_0^h\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz=-\dot{h} \tag{2.4}$$

Since u=v=0, at z=h, after taking the partial derivative operators outside the integral sign, we get, from (2.4),

$$\frac{\partial}{\partial x}\int_0^h u\,dz+\frac{\partial}{\partial y}\int_0^h v\,dz=-\dot{h} \tag{2.5}$$

Taking the partial derivative (2.5) with respect to time't',

$$\frac{\partial}{\partial x}+\int_0^h\frac{\partial}{\partial y}\int_0^h\frac{\partial v}{\partial t}dz=-\ddot{h}\text{ (second derivative)} \tag{2.6}$$

Multiplying (2.5) and (2.6) by μ and γ respectively and subsequently adding the resulting expressions we get,

$$\frac{\partial}{\partial x}\int_0^h\left(\mu u+\lambda\frac{\partial u}{\partial t}\right)dz+\frac{\partial}{\partial y}\int_0^h\left(\mu v+\lambda\frac{\partial v}{\partial t}\right)dz=-\left(\mu\dot{h}+\gamma\ddot{h}\right) \tag{2.7}$$

On using (2.2) and (2.3) in (2.7) we obtain the Reynolds's equation for second order fluid,

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{12}{h^3}\left(\mu \dot{h}+\gamma \ddot{h}\right) \tag{2.8}$$

We obtain the pressure distribution by solving equation (2.8) with the condition p =0 on the boundary. Thus,

$$p=\frac{12}{h^3}\left(\mu \dot{h}+\gamma \ddot{h}\right)(-1) \tag{2.9}$$

Where $s(x,y)=-1$

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial x^2}=-1$$

$$p=\frac{12}{h^3}\left(\mu \dot{h}+\gamma \ddot{h}\right)\left(s(x,y)\right) \tag{3}$$

And s=0 on the boundary.

Integration of the pressure given by (3) over the upper plate, we obtain the load,

$$W = F.S == \frac{A^2}{P}$$

$$S = \frac{A^2}{12\int\int s(x,y)\,dxdy} \tag{3.1}$$

4. Sinkage Relation

Sinkage relation between the film thicknesses 'h' and time't' for a given load W follows the equation (3.1) on integrating with initial condition.

We will transform (3.1) into dimensionless form:

$$SH^3 + K\frac{d^2H}{dT^2}+\frac{dH}{dT}=0 \tag{3.2}$$

where

$$\left.\begin{array}{l} H=\dfrac{h}{h_0}\text{(dimensionless film thickness)} \\ T=\dfrac{Wh_0t}{\mu A^2}\text{(dimensionless time)},\ K=\dfrac{Wh_0{}^2t}{\mu^2A^2} \end{array}\right\}$$

(Second order parameter)

And h_0 is the initial film thickness.

Solving (3.2) with initial condition T = 0,

$H = 1,\ \dfrac{dH}{dT}=-S$.

5. Elliptic Plates

Solving the Poisson's equation,

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}=-1,\ s=0 \text{ on } \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, \text{where } (e<1)$$

$$\text{therefore } s=\frac{1-\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)}{2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)} \tag{3.3}$$

satisfy $s_{xx}+s_{yy}=-1$

$$\text{Thus } \iint_{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1} s\,dx\,dy=\frac{\pi a^3 b^3}{4\left(a^2+b^2\right)}$$

The area $A=\pi ab$. Therefore we obtain the shape parameter from (3.1),

The Sinkage equation is

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+\frac{\Pi\left(2-e^2\right)}{3\sqrt{1-e^2}}H^3=0 \tag{3.4}$$

Case (ii): Circular plates

When a=b,

$$S=\frac{2\pi}{3}$$

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+\frac{2\Pi}{3}H^3=0 \tag{3.5}$$

6. Comparative Study

6.1. Equilateral Triangular Plates

Let the Poisson's Equation be

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}=-1$$

Keeping s=0 on the sides of the triangle we get

$$s=\frac{1}{2\sqrt{3}a}\left(x^2-3y^2\right)\left(\frac{\sqrt{3}}{2}a-x\right) \Rightarrow \iint s\ dx\ dy=\frac{\sqrt{3}}{320}a^4$$

We know that the area of the equilateral triangle is $A=\frac{\sqrt{3}}{4}a^2$

Hence we get the shape parameter as $S=\frac{5}{\sqrt{3}}$ and the Sinkage equation as

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+SH^3=0$$

6.2. Rectangular Plates

To get the shape parameter we solve the Poisson's Equation $\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}=-1$

With the boundary conditions

$s(x,0)=s(x,b)=s(0,y)=s(a,y)=0$

And get

$$s=\frac{ax-x^2}{2}-\frac{4a^2}{\pi^3}\sum_{1,3,5}^{\infty}\frac{\sinh\frac{n\pi x}{a}\cosh\frac{n\pi}{a}\left(y-\frac{b}{2}\right)}{n^3\cosh\frac{n\pi b}{2a}}$$

thus $\int_0^b\int_0^a s\,dx\,dy=\frac{a^3b}{12}-\frac{16a^4}{\pi^5}\sum_{1,3,5}^{\infty}\frac{\tanh\frac{n\pi b}{2a}}{n^5}$

Taking A=ab and solving we get the shape parameter as

$$S=\left[k-\frac{192k^2}{\pi^5}\sum_{1,3,5}^{\infty}\frac{\tanh\left(\frac{nx}{2k}\right)}{n^5}\right]^{-1}$$

The sinkage Equation in this case becomes

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+\frac{H^3}{k-\frac{192k^2}{\pi^5}\sum_{1,3,5}^{\infty}\frac{\tanh\left(\frac{nx}{2k}\right)}{n^5}}=0$$

6.3. Infinitely Long Rectangular Plates

This case cannot be obtained as like Finite rectangular plates as we used 'A' in non dimensionless form. In the present case A is infinite.

The Poisson's Equation takes the form $\frac{\partial^2 s}{\partial x^2}=-1, s=0$ at $x=\pm a$

This gives $s=\frac{1}{2}\left(a^2-x^2\right)$

Defining the dimensionless quantities

$H=\frac{h}{h_0}$ (dimensionless film thickness)

$T=\frac{Wh_0{}^2t}{8\mu a^3}, K=\frac{Wh_0{}^2\gamma}{8\mu^2a^3}$ (second order parameter)

And obtaining the pressure from the result (3)

The Load capacity per unit length of the plates is given by $W=\int_{-a}^{a}p\,dx=-\frac{8a^3}{h^3}\left(\mu\dot{h}+\gamma\ddot{h}\right)$

$$p=-\frac{6}{h^3}\left(\mu\dot{h}+\gamma\ddot{h}\right)$$

Thus the sinkage equation is obtained as follows:

$$\mu\dot{h}+\gamma\ddot{h}+\frac{Wh^3}{8a^3}=0$$

Using the dimensionless form in the above equation, the sinkage equation becomes

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+H^3=0$$

7. Results and Discussion

Hence, we obtained the sinkage relation between the film thicknesses and time in dimensionless form,

$$K\frac{d^2H}{dT^2}+\frac{dH}{dT}+SH^3=0$$

Where K denotes the second order parameter and s represents the shape parameter.

$S=1$ for infinitely long rectangular plates

$=\frac{2\pi}{3}$ for circular plates

$=\frac{\pi\left(2-e^2\right)}{3\sqrt{1-e^2}}$ for elliptic plates of eccentricity e

$=\frac{5}{\sqrt{3}}$ for equilateral triangular plates

$=\left[k-\frac{192k^2}{\pi^5}\sum_{1,3,5}^{\infty}\frac{\tan\left(\frac{n\pi}{2k}\right)}{n^5}\right]^{-1}$ for rectangular plates of side ratio k

Figure 2:

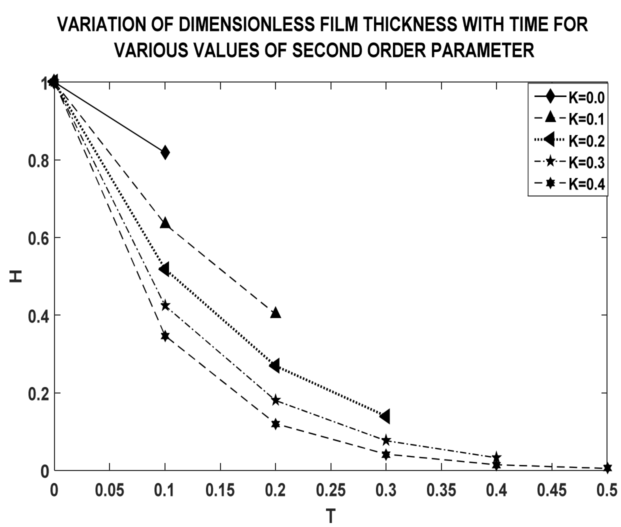


Figure 3:

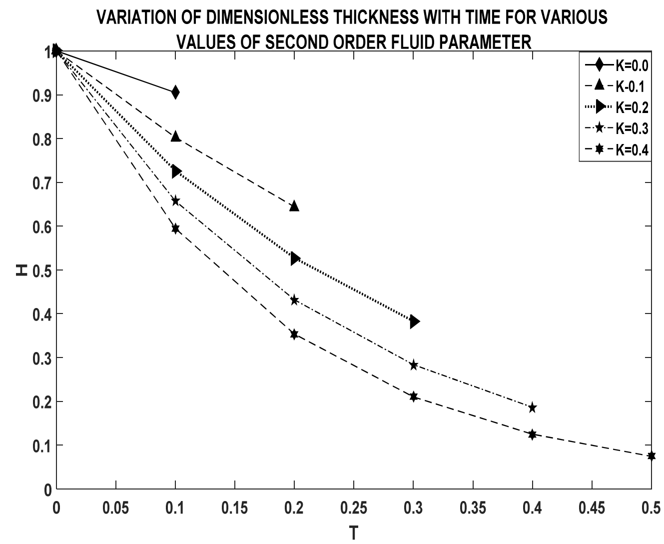
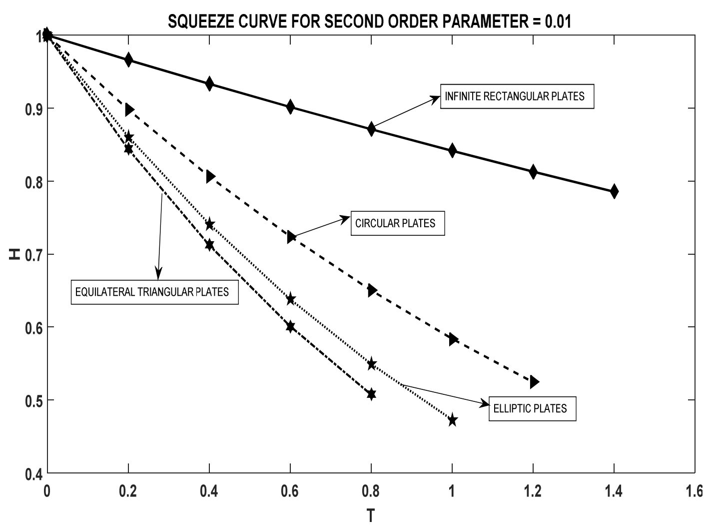


Figure 4:



From Fig. (2) It is shown that as the values of second order parameter increases the time increases. The dimensionless film thickness ranges from 0 to 1 and the film thickness of the bearing decreases with increase of time which shows the bearing (elliptic plates) characteristics.

From Fig. (3) It is shown that as the film thickness of the circular plates is increasing the time decreases for different values of second order parameter which shows the second order parameter satisfying the bearing.

From Fig. (4), for examination of squeeze times for various shapes, the crush bends for K=0.01. As in Newtonian liquids, press is logically quicker as we go from boundlessly long rectangular, circular plates, elliptic plates to equilateral triangular plates.

In the event that the plate begins from the position H=1, with the speed – S as on account of Newtonian liquid, we need to take the underlying condition as H=1, T=0.

Furthermore, the sinkage equation we got throughout the study is non linear differential equation, so there can't be any analytical solution possible. Here, we have used the numerical method "Runge Kutta method" with the assumption of step size 0.05.

A complete optimization have not been performed in this paper, but the range of plates considered was sufficient to suggest that the parameter used to draw these conclusions were not far from optimal for designing a bearing system which can hold the pressure generated by the fluid (Second order fluid). But as in Newtonian fluids, squeeze is progressively faster as we go from infinitely long rectangular, circular plates, Elliptic plates, to equilateral triangular plates. As we fix the second order parameter $K = 0.01$, the infinite rectangular plates shows more squeezing effect, the fluids expand and get shrink as the surface has enough space. So the normal and the tangential force have an equal effect on the surface which tends to have more squeezing effect. While in the case of equilateral triangular plates, the squeezing effect is less as it has a compact system.

The present approach involves two simplifications that will require more through examinations in future work.

- If we increase the step size for the second order parameter K from 0.01 to 0.05, we will get the clear description of the behavior of squeezing curve.

The sinkage curves for the straight line can be obtained from the property of Newtonian fluids, in Newtonian fluid, K=0, then the sinkage relation takes the form

$$s = \frac{1}{2T}\left(\frac{1}{H^2} - 1\right)$$

As K tends to ∞, the sinkage curves tends to straight line

$$s = \frac{1-H}{T}$$

This implies $\frac{1-H}{s} < T \le \frac{1}{2s}\left(\frac{1}{H^2} - 1\right)$

Essentially for a given time T, the dimensionless film thickness H is limited as $1 - sT < H \le \frac{1}{\sqrt{1+2sT}}$.

8. Conclusions

This paper considers the conduct/behavior of the liquid levels at the direction. Specifically, it centers on the sinkage degree (or the boundary condition for the absorbing/sink and reflecting/drift Cases). The set of second order equation is lessened to a set of first order equation using R-K technique. We reason that as we increment the estimation of second order parameter, the dimensionless film thickness of the plate diminishes, the time taken to diminishing the metal increments. Accordingly, the time taken to sink the object as the fluid flows an expansion which implies the bearing is having the ability to hold the weight produced by the fluid.

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