

Solving the Inexact Rough Intervals Vendor Selection Problems

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Abstract

This paper deals with vendor selection problem (VSP) in which a buyer allocates an order quantity for an item among a set of suppliers such that the required aggregate quality, services, and lead time requirements are achieved at minimum cost. The problem is considered by incorporating rough intervals in all of these characters adding to the allocated demand. A proposed procedure for solving the problem is suggested for obtaining optimal rough solution. A numerical example is given for illustration.

Keywords: Vendor selection problem; Rough interval; rough interval vendor selection; Uncertainty; Optimal rough solution

1. Introduction

Vendor selection problem (VSP) is an important process for an effective inventory management, supply chain management, business such as automobile manufacturing (Kumar et al. 2006), chemical industry (Crama et al. 2004) hospital (De Boer and Van der Wegen, 2003), telecommunications (Degraeve et al. 2005), constructions (De Carvalho and Costa, 2007), and also for a manufacturing firm (Degraeve and Roodhooft, 2000) because the cost and quality of goods and services. Sold are directly related to the cost and quality of goods and services purchased (Choi, and Hartley, 1996; Degraeve et al., 2000). Today's, a highly competitive and interrelated manufacturing environment, materials represent a sub-spatial part of the value of product. In view of the high percentage of the material cost, the key objective of the purchasing development ought to be purchasing the right quality of a product in the right quantity from the right source at the right time. The right source can provide the right quality of material on time at a reasonable price (Heizer and Render, 2001). Degraeve and Roodhooft, 2000 developed an activity based costing approach for determining the procurement strategies. Kumar et al. 2006 introduced a VSP incorporates three goals: cost minimization, quality maximization, and on- time- delivery maximization under the realistic constraints such as meeting the buyer's demand, vendor's capacity, vendor's quota flexibility, etc., and applied a fuzzy programming approach for treating the problem. Parthiban et al. 2013 introduced an integrated approach of multiple criteria decision making techniques such as fuzzy logic, strength- weakness- opportunity- threat analysis, and data envelopment analysis for the VSP. A several dimensions as: Price, delivery, quality,

and capacity which are important in the multiple objective vendor selection decision are analyzed by Dickson, 1966. Aamer and Sawhney, 2004 are introduced different approaches for solving VSP. Agakishiyev, 2016 studied the supplier selection under deterministic, stochastic, interval-valued and fuzzy information. Zadeh et al. 2017 treat a supplier selection problem under certainty, uncertainty, and stochastic conditions through three techniques: deterministic data envelopment analysis (DEA), fuzzy DEA, and stochastic DEA. Khalifa, 2017 studied vendor selection problem with fuzzy parameters both in a price of a unit item, upper limit of the quantity available, and an aggregate demand of the item through an interactive fuzzy programming approach. He et al. 2009 developed a class of special chance-constrained programming models and designed genetic algorithm for the VSP. Based on DEA and their combination, Kontis and Vrysagotis, 2011 introduced a literature review of multi-criteria decision making approaches for evaluation and selection supplier. Xia and Wu, 2007 introduced two types of supplier selection problems as single and multiple sourcing. Diaz-Madronero et al. 2010 introduced VSP with fuzzy goals, and suggested an interactive method for solving fuzzy multi-objective VSP having fuzzy data represented by S-curve membership functions. Based on hesitant fuzzy sets, Zhou et al. 2018 present a preference model for supplier selection involving the cost and service quality that is to select the suppliers. Song et al. 2018 proposed a method for selecting a green supplier in a dynamic environment. Under uncertainty on the selection of the suppliers within the framework of binary programming, Ekhtiari et al. 2018 proposed a nadir compromise programming model for decision-making. Arikan, 2013 proposed an interactive approach for solving multiple sourcing supplier selection problem presented as a multiple objective linear programming with fuzzy demand level and / or fuzzy aspiration levels of objectives.

In this paper, a vendor selection problem (VSP) having rough interval in all of cost, aggregate quality, services, lead time, and allocated demand is investigated. A proposed procedure of the problem is suggested for obtaining an optimal rough solution.

The remainder of the paper is organized as in the following sections: In section 2; some preliminaries need in the paper are presented. In section 3, a fully rough intervals vendor selection problem is introduced as specific definition and properties. In section 4, a solution procedure for solving the problem is suggested. In section 5, an illustrative numerical example is given to clarify the obtained results. Finally some concluding remarks are reported in section 6.

2. Preliminaries

In order to discuss our problem conveniently, we shall state some necessary results on rough intervals arithmetic operations (Lu et al. 2011).

Definition 1. Let x denote a compact set of real numbers. A rough interval x^R is defined as:

$$x^R = [x^{(UAI)} : x^{(LAI)}] \quad (1)$$

Where, $x^{(UAI)}$ and $x^{(LAI)}$ are upper and lower approximation intervals of x^R , respectively.

Let $RI(\mathfrak{R}) = \{[a^{(UAI)} : a^{(LAI)}] : a^{(LAI)} \subseteq a^{(UAI)}, a^{(LAI)} \subseteq \mathfrak{R} = (-\infty, \infty), a^{(UAI)} \subseteq \mathfrak{R}\}$ be the set of all rough intervals on \mathfrak{R} .

Definition 2. Let $* \in \{+, -, \times, /\}$ be a binary operations on rough intervals. For rough intervals x^R and y^R , when $x^R \geq 0$ and $y^R \geq 0$ we have:

$$x^R (+) y^R = [x^{(UAI)} + y^{(UAI)} : x^{(LAI)} + y^{(LAI)}], \quad (2)$$

$$x^R (-) y^R = [x^{(UAI)} - y^{(LAI)} : x^{(LAI)} - y^{(UAI)}], \quad (3)$$

$$x^R (\times) y^R = [x^{(UAI)} \times y^{(UAI)} : x^{(LAI)} \times y^{(LAI)}], \quad (4)$$

$$x^R (/) y^R = [x^{(UAI)} / y^{(UAI)} : x^{(LAI)} / y^{(LAI)}]. \quad (5)$$

As, $x^{(UAI)}$, $x^{(LAI)}$, $y^{(UAI)}$, and $y^{(LAI)}$ are conventional intervals, the equations (2)- (5) can be transferred to the following functions if we letting $x^{(UAI)} = [x^{-(UAI)}, x^{+(UAI)}]$, $y^{(UAI)} = [y^{-(UAI)}, y^{+(UAI)}]$, (2)- (5) can be rewritten as follows: $x^{(LAI)} = [x^{-(LAI)}, x^{+(LAI)}]$, $y^{(LAI)} = [y^{-(LAI)}, y^{+(LAI)}]$,

$$1. x^R(+)y^R = [[x^{-(UAI)} + y^{-(UAI)}, x^{+(UAI)} + y^{+(UAI)}] : [x^{+(LAI)} + y^{+(LAI)}, x^{-(LAI)} + y^{-(LAI)}]] \tag{6}$$

$$2. x^R(-)y^R = [[x^{-(UAI)} - y^{+(UAI)}, x^{+(UAI)} - y^{-(UAI)}] : [x^{-(LAI)} - y^{+(LAI)}, x^{+(LAI)} - y^{-(LAI)}]] \tag{7}$$

$$3. x^R(\times)y^R = [[x^{-(UAI)} \times y^{-(UAI)}, x^{+(UAI)} \times y^{+(UAI)}] : [x^{-(LAI)} \times y^{-(LAI)}, x^{+(LAI)} \times y^{+(LAI)}]] \tag{8}$$

$$4. x^R(\div)y^R = [[x^{-(UAI)} \div y^{+(UAI)}, x^{+(UAI)} \div y^{-(UAI)}] : [x^{-(LAI)} \div y^{+(LAI)}, x^{+(LAI)} \div y^{-(LAI)}]] \tag{9}$$

Definition3. A function $Z : \mathfrak{R}^n \rightarrow RI(\mathfrak{R})$ is said to be a rough interval function (because $Z(x)$ is a rough interval in \mathfrak{R}). Similarly, we denote the rough interval function Z with the following objective $Z(x) = [Z^{(UAI)}(x) : Z^{(LAI)}(x)]$, where for every $x \in \mathfrak{R}^n$, $Z^{(UAI)}$, $Z^{(LAI)}$ are the real upper and lower approximation intervals and $x^{(LAI)} \subseteq x^{(UAI)}$.

Definition4. To interpret the meaning of optimizing of rough interval, we introduce the partial order relation:

Let $x^R = [[x^{+(UAI)}, x^{-(UAI)}] : [x^{+(LAI)}, x^{-(LAI)}]]$, and $y^R = [[y^{+(UAI)}, y^{-(UAI)}] : [y^{+(LAI)}, y^{-(LAI)}]]$ be two rough intervals, then we say that:

$$x^R (\leq) y^R \text{ if and only if } x^{+(UAI)} \leq y^{+(UAI)}, \text{ and } x^{-(UAI)} \leq y^{-(UAI)} \tag{10}$$

$$x^R (<) y^R \text{ if and only if } x^R (\leq) y^R, \text{ and } x^R \neq y^R. \tag{11}$$

3. Problem Formulation and Solution Concepts

In this section, consider the linear programming model of the price minimizing problem constrained with performance measures of quality, services, and lead time and for the sake of the completeness the model was provided as (Pan, 1989) with fully rough intervals

Model1: (FR- MOLP) $\min Z(x^R, p^R) = \sum_i (p_i)^R (x_i)^R$

Subject to

$$\sum_i (q_i)^R (x_i)^R \geq Q^R,$$

$$\sum_i (s_i)^R (x_i)^R \geq S^R,$$

$$\sum_i (l_i)^R (x_i)^R \leq L^R,$$

$$\sum_i (x_i)^R = 1,$$

$$(x_i)^R \geq 0^R; \forall i.$$

Where, for a given vendor i , $(p_i)^R$ is the price of the item, $(q_i)^R$ is the quality level, $(s_i)^R$ is the service level, $(l_i)^R$ is the lead time, whereas Q^R is the overall quality level required, S^R is the overall service level required, L^R is the overall lead time required, and $(x_i)^R$ is the fraction of demand allocated to vendor i . It is clear that all of $(p_i)^R, (q_i)^R, (s_i)^R, (l_i)^R, Q^R, S^R, L^R, (x_i)^R$ belong to

$F(\mathfrak{R})$ which denoted as the set of all rough intervals on R , that is , for any $F(\mathfrak{R})$, f satisfies : $f^R = [f^{(UAI)}; f^{(LAI)}]$. It is noted that: $(p_i)^R = [[p_i^{-(UAI)}, p_i^{+(UAI)}]:[p_i^{-(LAI)}, p_i^{+(LAI)}]]$, $(q_i)^R = [[q_i^{-(UAI)}, q_i^{+(UAI)}]:[q_i^{-(LAI)}, q_i^{+(LAI)}]]$, $(s_i)^R = [[s_i^{-(UAI)}, s_i^{+(UAI)}]:[s_i^{-(LAI)}, s_i^{+(LAI)}]]$, $(l_i)^R = [[l_i^{-(UAI)}, l_i^{+(UAI)}]:[l_i^{-(LAI)}, l_i^{+(LAI)}]]$, $Q^R = [[Q^{-(UAI)}, Q^{+(UAI)}]:[Q^{-(LAI)}, Q^{+(LAI)}]]$, $S^R = [[S^{-(UAI)}, S^{+(UAI)}]:[S^{-(LAI)}, S^{+(LAI)}]]$, $L^R = [[L^{-(UAI)}, L^{+(UAI)}]:[L^{-(LAI)}, L^{+(LAI)}]]$. Moreover $(x_i)^R$ may be written as $(x_i)^R = [[x_i^{-(UAI)}, x_i^{+(UAI)}]:[x_i^{-(LAI)}, x_i^{+(LAI)}]]$

Observe that $\mathfrak{R} \subset I(\mathfrak{R}) \subset F(\mathfrak{R})$.

The FR- MOLP can be rewrite as follows:

Model2: Find $(x_i)^{R*}$ from M^R such that:

$$\sum_i (p_i)^R (x_i)^{R*} = \min_{(x_i)^R \in M^R} \sum_i (p_i)^R (x_i)^R \quad (12)$$

Where, $M^R = \left\{ \begin{aligned} &(x_i)^R : \sum_i (q_i)^R (x_i)^R \geq Q^R; \sum_i (s_i)^R (x_i)^R \geq S^R, \\ &\sum_i (l_i)^R (x_i)^R \leq L^R; \sum_i (x_i)^R = 1; (x_i)^R \geq 0^R; \forall i. \end{aligned} \right\}$, and min means that \wedge can be

valued in $\left\{ \sum_i (p_i)^R (x_i)^R : (x_i)^R \in M^R \right\}$.

Definition5.(Rough optimal solution). The $(x_i)^{R*}$ which satisfies the condition in (12) is called a rough optimization solution of Model 2.

Corresponding to Model2, the following sub models are structured as:

Model2.1: Find $(x_i)^{+(UAI)*}$ from $M^{+(UAI)}$ such that:

$$\sum_i (p_i)^{+(UAI)} (x_i)^{+(UAI)*} = \min_{(x_i)^{+(UAI)} \in M^{+(UAI)}} \sum_i (p_i)^{+(UAI)} (x_i)^{+(UAI)} \quad (13)$$

Model2.2: Find $(x_i)^{+(LAI)*}$ from $M^{+(LAI)}$ and $(x_i)^{+(LAI)} \leq (x_i)^{+(UAI)*}$ such that:

$$\sum_i (p_i)^{+(LAI)} (x_i)^{+(LAI)*} = \min_{\substack{(x_i)^{+(LAI)} \in M^{+(LAI)} \\ (x_i)^{+(LAI)} \leq (x_i)^{+(UAI)*}} \sum_i (p_i)^{+(LAI)} (x_i)^{+(LAI)} \quad (14)$$

Model2.3: Find $(x_i)^{-(LAI)*}$ from $M^{-(LAI)}$ and $(x_i)^{-(LAI)} \leq (x_i)^{+(LAI)*}$ such that:

$$\sum_i (p_i)^{-(LAI)} (x_i)^{-(LAI)*} = \min_{\substack{(x_i)^{-(LAI)} \in M^{-(LAI)} \\ (x_i)^{-(LAI)} \leq (x_i)^{+(LAI)*}} \sum_i (p_i)^{-(LAI)} (x_i)^{-(LAI)} \quad (15)$$

Model 2.4: Find $(x_i)^{-(UAI)*}$ from $M^{-(UAI)}$ and $(x_i)^{-(UAI)} \leq (x_i)^{-(LAI)*}$ such that:

$$\sum_i (p_i)^{-(UAI)} (x_i)^{-(UAI)*} = \min_{\substack{(x_i)^{-(UAI)} \in M^{-(UAI)} \\ (x_i)^{-(UAI)} \leq (x_i)^{-(LAI)*}} \sum_i (p_i)^{-(UAI)} (x_i)^{-(UAI)} \quad (16)$$

Suppose the rough optimal solutions of Model 2.1,Model 2.2,Model 2.3, and Model 2.4 are

$$(x_i)^{+(UAI)*}; Z^{+(UAI)*}, (x_i)^{+(LAI)*}; Z^{+(LAI)*}, (x_i)^{-(LAI)*}; Z^{-(LAI)*}, (x_i)^{-(UAI)*}; Z^{-(UAI)*}.$$

Then the optimal rough solution of Model 1 as follows

$$\min Z^R = [[Z^{-(UAI)}, Z^{+(UAI)}]:[Z^{-(LAI)}, Z^{+(LAI)}]]$$

$$\begin{bmatrix} (x_1)^R \\ (x_2)^R \\ \vdots \\ (x_n)^R \end{bmatrix} = \begin{bmatrix} \left[\left[(x_1)^{-(UAI)}, (x_1)^{+(UAI)} \right] : \left[(x_1)^{-(LAI)}, (x_1)^{+(LAI)} \right] \right] \\ \left[\left[(x_2)^{-(UAI)}, (x_2)^{+(UAI)} \right] : \left[(x_2)^{-(LAI)}, (x_2)^{+(LAI)} \right] \right] \\ \vdots \\ \left[\left[(x_n)^{-(UAI)}, (x_n)^{+(UAI)} \right] : \left[(x_n)^{-(LAI)}, (x_n)^{+(LAI)} \right] \right] \end{bmatrix} \tag{17}$$

This optimal rough solution is a rough internal solution, for the decision maker it contains more information.

Theorem1. If $(x_i)^{+(UAI)*}$, $(x_i)^{+(LAI)*}$, $(x_i)^{-(UAI)*}$, and $(x_i)^{-(LAI)*}$ are optimal rough solutions of Model 2.1, Model 2.2, Model 2.3, and Model 2.4; respectively, then $x^{R*} = \left[\left[(x_i)^{-(UAI)*}, (x_i)^{+(UAI)*} \right] : \left[(x_i)^{-(LAI)*}, (x_i)^{+(LAI)*} \right] \right]$ is a non dominated rough solution of Model 2.

Proof: (As similar to Wu, 2008).

4. Solution Procedure

In this section, a solution procedure for solving the (FR- MOLP) is considered as in the following steps:

Step1: Convert Model 1 into two boundaries Z^{+R} , and Z^{-R} , respectively;

Step2: According to the lower and upper approximation intervals, each boundary model in step1 may be transformed into the corresponding two submodels;

Step3: Firstly, solve the first upper bound Model2.1 to obtain $(x_i)^{+(UAI)*}$, and the corresponding optimum value $Z^{+(UAI)*}$;

Step4: Solve the Model 2.2 after Adding the constraint $(x_i)^{+(LAI)} \leq (x_i)^{+(UAI)*}$ to the second upper bound submodel), to obtain $(x_i)^{+(LAI)*}$, and the corresponding $Z^{+(LAI)*}$;

Step5: Solve the Model 2.3 after Adding the constraint $(x_i)^{-(LAI)} \leq (x_i)^{+(LAI)*}$ to the first lower bound submodel), to obtain $(x_i)^{-(LAI)*}$, and the corresponding $Z^{-(LAI)*}$;

Step6: Solve the Model 2.4 after Adding the constraint $(x_i)^{-(UAI)} \leq (x_i)^{-(LAI)*}$ to the second lower bound submodel), to obtain $(x_i)^{-(UAI)*}$, and the corresponding $Z^{-(UAI)*}$;

Step7: Incorporate all the solutions of models: Model 2.1, Model2.2, Model 2.3, and Model 2.4 to obtain the optimal rough solution of Model2 as

$$\begin{aligned} Z^R &= \left[\left[Z^{-(UAI)}, Z^{+(UAI)} \right] : \left[Z^{-(LAI)}, Z^{+(LAI)} \right] \right], \text{ and} \\ (x_i)^R &= \left[\left[(x_i)^{-(UAI)}, (x_i)^{+(UAI)} \right] : \left[(x_i)^{-(LAI)}, (x_i)^{+(LAI)} \right] \right] \forall i = 1, 2, \dots, n \end{aligned} \tag{18}$$

5. Numerical Example

Consider the following problem

Table 1: Rough interval mathematical model

Factor Required Supplier	-----	A
Quantity	0- 1	[1: 1]
Price (\$/ unit)	[[9.6, 10.4]: [9.8, 10.2]]	-----
Quality (%)	[[85.0, 91.0]: [87.0, 89.0]] [[88.0, 95.0]: [90.0, 93.0]]	
Lead time (days)	[[24.0, 30.0]: [26.0, 29.0]]	[[20.0, 25.0]: [22.0, 24]]
Service (%)	[[83.0, 90.0]: [85.0, 88.0]]	[[83.0, 90.0]: [85.0, 88.0]]

Table 2: Rough interval mathematical model

Factor Required Supplier	-----	B
Quantity	0- 1	[1: 1]
Price (\$/ unit)	[[9.5, 10.5]: [9.7, 10.3]]	-----
Quality (%)	[[87.0, 94.0]: [89.0, 92.0]]	[[88.0, 95.0]: [90.0, 93.0]]
Lead time (days)	[[20.0, 25.0]: [22.0, 24.0]]	[[20.0, 25.0]: [22.0, 24]]
Service (%)	[[85.0, 95.0]: [87.0, 93.0]]	[[83.0, 90.0]: [85.0, 88.0]]

Table 3: Rough interval mathematical model

Factor Required Supplier	-----	C
Quantity	0- 1	[1: 1]
Price (\$/ unit)	[[8.0, 11.0]: [9.0, 10.0]]	-----
Quality (%)	[[83.0, 98.0]: [85.0, 95.0]]	[[88.0, 95.0]: [90.0, 93.0]]
Lead time (days)	[[21.0, 25.0]: [22.0, 23.0]]	[[20.0, 25.0]: [23.0, 24]]
Service (%)	[[84.0, 96.0]: [87.0, 94.0]]	[[83.0, 90.0]: [85.0, 88.0]]

Table 4: Optimal rough solution

No. of Model	Model2.3	Model2.4
Optimal rough solution	$(x_1)^{+(UAI)*} = 0$ $(x_2)^{+(UAI)*} = 0$ $(x_3)^{+(UAI)*} = 1$	$(x_1)^{-(UAI)*} = 0$ $(x_1)^{-(UAI)*} = 0$ $(x_1)^{-(UAI)*} = 1$
Optimal rough solution:	$(x_1)^{R*} = 0, (x_2)^{R*} = 0, (x_3)^{R*} = 1$	
Optimum rough solution:	$Z^{R*} = [[10 \quad 11]: [10, \quad 10.5]]$	

6. Concluding Remarks

In this paper, the inexact rough intervals vendor selection problem has been investigated. The advantages are that vendor selection problem with rough intervals allows the DM to deal with a situation realistically. A numerical example has been given to illustrate the proposed approach which has been investigated for obtaining optimal rough solution.

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References

- [1] Aamer, A. M., and Sawhney, R. (2004). Review of supplier from a production perspective. In IIE Annual Conference and Exhibition Conference Proceedings, 2135- 2140.
- [2] Agakishiyev, E. (2016). Supplier selection problem under z-information. *Procedia Computer Science*, (102): 418- 425.
- [3] Arikan, AF. (2013). A fuzzy solution approach for multi- objective supplier selection. *Expert Systems with Application*, (40): 947- 952.
- [4] Crama, Y., Pascual, R. J., and Torres, A. (2004). Optimal procurement decisions in the presence of total quality discounts and alternative product recipes. *European Journal of Operational Research*, (159): 364- 378.
- [5] Choi, T. Y., Hartley, J. L. (1996). An exploration of supplier selection practices across the supply chain. *Journal of Operations Research*, 14(4): 333- 343.
- [6] De Boer, L., and Van der Wegen, L. L. M. (2003). Practice and promise of formal supplier selection: A study of four empirical cases. *Journal of Purchasing and Supply Management*, (9): 109- 118.
- [7] Degraeve, Z., and Roodhif, F. (2000). A mathematical programming approach for procurement using activity based costing. *Journal of Business Finance& Accounting*, (27): 69- 98.
- [8] Degraeve, Z., Labro, E., and Roodhif, F. (2000). An evaluation of vendor selection models from a total cost of ownership perspective. *European Journal of Operational Research*, (125): 34- 58.
- [9] Degraeve, Z., Labro, E., and Roodhooft, F. (2005). Constructing a total cost of ownership supplier selection methodology based on activity- based costing and mathematical programming. *Accounting and Business Research*, 35(1): 3- 27.
- [10] Diaz- Madronero, M, Peidro, D., and Vasant, P. (2010). Vendor selection problem by using an interactive fuzzy multi- objective approach with modified S- curve membership functions. *Computer Mathematics with Applications*, (60): 1038- 1048.
- [11] Dickson, G. W. (1966). An analysis of vendor systems and decisions. *Journal of Purchasing*, 2(1): 1-5.
- [12] Ekhtiari, M., Zandieh, M., Alem- Tabriz, A., and Rabieh, M. (2018). A nadir compromise programming for supplier selection problem under uncertainty. *International Journal of Industrial Engineering& Production*, 29(1): 1-14.
- [13] He, S., Chaudhry, S. S., Lei, Z., and Baohua, W.(2009). Stochastic vendor selection problem: Chance- constrained model and genetic algorithms. *Annals of Operations Research*, (168): 169- 179.
- [14] Heizer, J., and Render, B. (2001). *Operations Management*. Prentice- Hall, 431- 457.
- [15] Khalifa, A.H. (2017)). Interactive fuzzy programming approach for vender selection problem in supply chain with fuzzy parameters. *Fuzzy Mathematics*, 25(4): 239-250
- [16] Kontis, P- a., and Vrysagotis, V. (2011). Supplier selection problem: A literature review of multi- criteria decision- making approach based on DEA. *Advances in Management& Applied Economics*, 1(2): 207- 219.
- [17] Kumar, M., Vrat, P., and Shankar, R. (2006). A fuzzy programming approach for vendor selection problem in a supply chain. *International Journal of Production Economics*, 101(2): 273- 285.
- [18] Lu, H., Huang, G., and He, L. (2011). An inexact rough- interval fuzzy linear programming method for generating conjunctive water- allocation strategies to agriculture irrigation systems. *Applied Mathematical Modeling*, (35): 4330- 4340.
- [19] Pan, A. C. (1989). Allocation of order quantities among suppliers. *Journal of Purchasing and Materials Management*, 25(2): 36- 39.
- [20] Parthiban, P., Abdul Zubar, H., and Katarak, P.(2013). Vendor selection problem: multicriteria on strategic decisions. *International Journal of Production Research* 51(5): 1535- 1548.

- [21] Song, W., Chen, Z., Liu, A., Zhu, Q., Zhao, W., and Tsai, S. B. (2018). A study on green supplier selection in dynamic environment. *Sustainability*, (10): 1226- 1247.
- [22] Wu, H. C. (2008). On interval- valued nonlinear programming problems. *Journal of Mathematical Analysis and Applications*, 338(1): 299- 316.
- [23] Xia, W., and Wu, Z. (2007). Supplier selection with multiple criteria in volume discount environments. *Omega the international Journal of Management Science*, (35): 494- 504.
- [24] Zadeh, A. A., Rahimi, Y., Zarri, M., Ghaderi, A., and Shabanpour, N. (2017). A decision-making methodology for vendor selection problem with uncertain inputs. *The International Journal of Transportation Research*, 9(3): 123- 140.
- [25] Zhou, Z., Dou, Y., Liao, T., and Tan, Y. (2018). A preference model for supplier selection based hesitant fuzzy sets. *Sustainability*, (10): 659-671.