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Semi-analytical Theory of the Mean Orbital Motion due to the Effect of Gravitational Waves

M. H. A. Youssef
Astronomy Department, Faculty of Science, Cairo University, Egypt
E-mail: mhyoussef@sci.cu.edu.eg

Abstract

We have developed a semi- analytical theory based on the concept of filtered elements permitting the exact separation between short period and long period variations of the orbital motion. The characteristic of the method concern the use of an analytical procedure based on Lie-transforms and a numerical integration of the orbit averaged perturbations due to the effect of gravitational waves. Two successive canonical transformations are used to obtain the analytical solution for third order effect in short-period, long- period and secular perturbations of orbital elements. The long-period terms will be never eliminated from the Hamiltonian of gravitational waves, consequently the perturbation of gravitational waves is purely of short-period, and decreasing with time. The short-period terms are obtained numerically for four different kinds of orbits. The gravitational waves will be acted on the plane of the orbit but there is no effect on the shape or the size of the orbit.

Keywords: gravitational waves; general relativity; celestial mechanics; astrodynamic; perturbation; Lie transform.

1. Introduction

Einstein showed that weak field solutions of his field equations lead to the wave equation. Nowadays there is strong consensus among physicists that gravitational waves do exist and that the real questions concern only the sources and strengths of the radiation .Nevertheless the question of the existence of gravitational radiation remains open until it is discussed experimentally and its properties (such as the velocity of propagation and the states of polarization) are determined. Extensive efforts have been made to detect it experimentally. The new generation of ground-based laser interferometric detectors such as LIGO, VIRGO, GEO600, TAMA300 and ACIGA categorized gravitational waves sources by the frequency band and the method for detecting (McCleland and et al, 2000). The motions of celestial objects in space are usually governed by the mutual attraction forces between them, but there exist other external forces or external effects in space. These effects cause perturbation in the motion of the objects in the forms of variation in coordinates or in the Keplerian elements or canonical variables describing the orbit. One of these external effects is the gravitational wave from a burst or from a cosmological origin. This effect has been investigated from many authors , Mashoon (Mashoon , 1978,1987) worked with cylindrical coordinates to find the first order orbital variations, Nelson and Chou (Nelson and Chou ,1982) formed the equations of variation of coordinates in an elliptic orbit and solved them numerically, Rudenko (Rudenko ,1975) discussed the interaction of GW with circular orbit of an artificial satellite. Ivashchenko (Ivashchenko ,1987) used Lagrange’s planetary equations to find a first order solution in all the elements, and (Anderson, 1971). The aim of the present work is to evaluate the perturbations produced by a weak gravitational wave on the canonical variables using the perturbation technique of the canonical Lie-transforms. One canonical transformation is used to derive analytical expressions of Short-period terms of an elliptic orbit of any earth's orbiter. The solution is expressed in terms of the Delaunay variables up to order three in eccentricity, and solving the equations of motion numerically to filtrate the short-period and long-period perturbations of the orbital elements due to the effect of gravitational waves.

2. Mathematical Formulations

All methods developed to detect gravitational waves depend more or less on the fact that maximum variation on the separation of two particles, produced by a gravitational wave propagating along the Z axis, occurs if the particles are located in the XY plane. For a plane wave travelling in the Z direction the only non-vanishing components are $h_{11} = -h_{22}$ and $h_{12} = h_{21}$, so the metric describing the wave becomes

$$ds^2 = c^2\, dt^2 - dz^2 - (1 - h_{11})\, dx^2 - (1 + h_{11})\, dy^2 + 2h_{12}\, dx\, dy \tag{1}$$

And the separation

$$dl^2 = dz^2 + (1 - h_{11})\, dx^2 + (1 + h_{11})\, dy^2 + 2h_{12}\, dx\, dy \tag{2}$$

Thus a plane gravitational wave is determined by two quantities $h_{11} = -h_{22}$ and $h_{12} = h_{21}$, so the gravitational waves are transverse waves whose polarization is determined by a symmetric tensor of the second rank in the XY plane, the sum of whose diagonal terms $h_{11} + h_{22}$ is zero, and can be written as the sum of two components

$$h_{11} = h_+ \cos(n_g t + \alpha_1)\ , \quad h_{12} = h_x \cos(n_g t + \alpha_2) \tag{3}$$

Where, n_g is the frequency of the wave. α_1 and α_2 are the phase difference , h_+ and h_x are the amplitudes of the wave in the two orthogonal directions in the transverse plane . The radial acceleration (tidal forces) components produced by the plane wave will obtain via the equation of geodesic deviation (Straumann, 1984)

$$\frac{d^2 n^i}{dt^2} = \frac{1}{2}\frac{\partial^2 h_{ij}}{\partial t^2}\, n^j \tag{4}$$

Where n is the separation vector between neighboring geodesics, in terms of X, Y, Z coordinates, equation (4) yields

$$\begin{aligned}
\frac{d^2 x}{dt^2} &= \frac{1}{2}\frac{\partial^2 h_{11}}{\partial t^2} x + \frac{1}{2}\frac{\partial^2 h_{12}}{\partial t^2} y \\
\frac{d^2 y}{dt^2} &= \frac{1}{2}\frac{\partial^2 h_{12}}{\partial t^2} x - \frac{1}{2}\frac{\partial^2 h_{11}}{\partial t^2} y \\
\frac{d^2 z}{dt^2} &= 0
\end{aligned} \tag{5}$$

Or

$$\begin{aligned}
F_x &= h_1 x + h_2 y \\
F_y &= h_2 x - h_1 y \\
F_z &= 0
\end{aligned} \tag{6}$$

Where

$$h_1 = \frac{1}{2}\frac{\partial^2 h_{11}}{\partial t^2} \quad \text{and} \quad h_2 = \frac{1}{2}\frac{\partial^2 h_{12}}{\partial t^2} \tag{7}$$

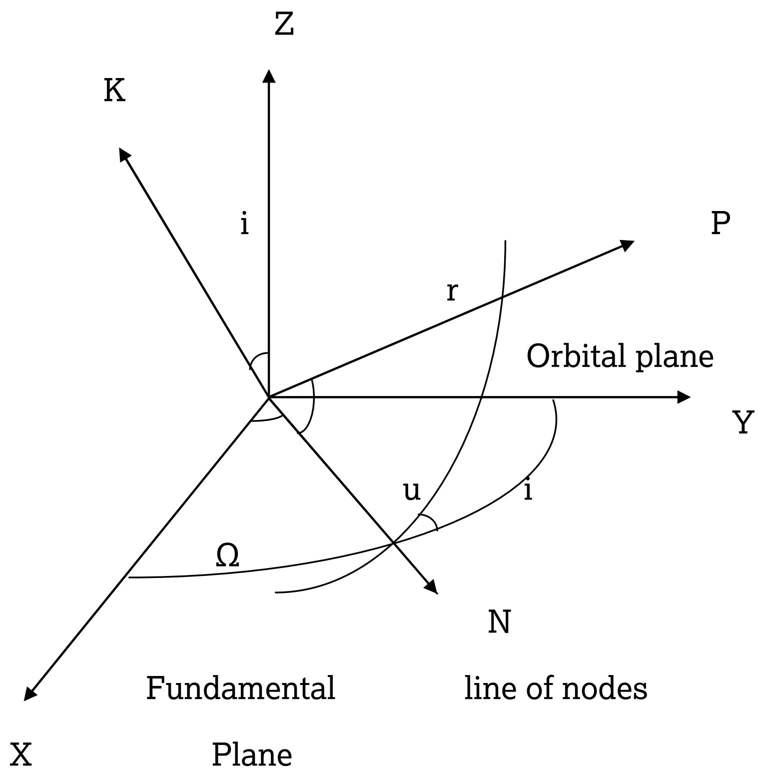
The coordinates x, y, and z of the moving celestial object in the inertial frame XYZ in terms of the elements of the orbit (a, e, I, ω, Ω, M) as in fig. (1), are

$$\begin{aligned}
x &= r\,(\cos\Omega \cos u - \sin\Omega \sin u \cos i) \\
y &= r\,(\sin\Omega \cos u + \cos\Omega \sin u \cos i) \\
z &= r \sin u \sin i
\end{aligned} \tag{8}$$

Where, i is the orbital inclination on the fundamental plane, Ω is the angle of longitude and u is equal the sum of $\omega + f$, the argument of periapsis ω and the true anomaly f as in fig.(1).The elements at a specified instant t_0 (say) represent the osculating elements of the osculating orbit. This orbit is tangent to the true orbit (or the actual orbit) which represents the trajectory or the path of the moving body under the actual forces. At another instant t_1 (say) the elements at the two instants represent the perturbations in the element during the interval ($t_1 - t_0$), or the perturbation in canonical variables. The Delaunay set of canonical variables (l, g, h, L, G, H) are defined in terms of Keplerian elements (a, e, i, Ω, M) by

$$\begin{aligned} l &= M = n\,(t - \tau) \quad &;\quad L &= \sqrt{\mu a} \\ g &= \omega \quad &;\quad G &= \sqrt{\mu a}(1 - e^2) \\ h &= \Omega \quad &;\quad H &= G \cos i \end{aligned} \tag{9}$$

Figure 1: The position vector of moving celestial object in XYZ



Where, M is the mean anomaly, a the semi-major axis of the orbit of moving celestial object, e the eccentricity, n the mean motion of the body in the orbit, and τ the time of periapsis passage. μ is the reduced mass of earth. Thus equation (8) in terms of canonical variables will be

$$\begin{aligned} x &= r\,(\cos h \cos (g + f) - \sin h \sin (g + f) \cos i) \\ y &= r\,(\sin h \cos (g + f) + \cos h \sin (g + f) \cos i) \\ z &= r \sin (g + f) \sin i \end{aligned} \tag{10}$$

From the components of the acceleration vector at a point with x, y, z in equation (5), we can construct the Hamiltonian of GW in the form

$$H = H_0 + \epsilon H_1$$

$$H_0 = -\frac{\mu^2}{2L^2} \tag{11}$$

$H_1 = r^2 \{ \frac{s^2}{8} [\cos(2h + n_9 t + \alpha_1) - \cos(2h - n_9 t - \alpha_1) - A_g [\sin(2h + n_9 t + \alpha_2) + \sin(2h - n_9 t - \alpha_2)]] + (\frac{c^2}{16} + \frac{c}{8} - \frac{1}{16})[\cos(2f + 2g + 2h + n_9 t + \alpha_1) + \cos(2f + 2g + 2h -$ n9 t− α1 − *Ag* [sin2f+2g+2h+ n9 t+ α2 + sin2f+2g+2h− n9 t− α2 +sin2f+2g−2h+ n9 t+ α2 + sin2f+2g−2h− n9 t− α2]]+ *(c216− c8+ 116)* [cos2f+2g−2h+ n9 t+ α1 + cos2f+2g−2h− n9 t− α1]} (12)

Where, H_0 is unperturbed Hamiltonian from the solution of Hamilton-Jacobi equation of two-body system and H_1 is the perturbed Hamiltonian due to the effect of incident plane gravitational wave on the elliptic orbit of the moving celestial object. The Hamiltonian depends explicitly on the time through the term $n_9 t$. To remove the time from the Hamiltonian we set $n_9 t = l_4$ and augment the Delaunay variables by the pair (l_4 , L_4) , and $L_4 = -\frac{\partial H}{\partial l_4}$, so

$$l_4 = n_9 t \qquad ; \qquad L_4 = -\frac{\partial H}{\partial l_4} \tag{13}$$

Therefore the Hamiltonian which describes the relative motion of the celestial object under the effect of GW can expressed as

$$\begin{aligned} H = & -\frac{\mu^2}{2L^2} + n_9 L_4 + \epsilon L^4 \frac{r^2}{a^2} \{\frac{s^2}{8} [\cos(2h + l_4 + \alpha_1) - \cos(2h - l_4 - \alpha_1) - \\ & A_g [\sin(2h + l_4 + \alpha_2) + \sin(2h - l_4 - \alpha_2)]] + (\frac{c^2}{16} + \frac{c}{8} - \frac{1}{16})[\cos(2f + 2g + 2h + l_4 + \\ & \alpha_1) + \cos(2f + 2g + 2h - l_4 - \alpha_1) - A_g [\sin(2f + 2g + 2h + l_4 + \alpha_2) + \sin(2f + 2g + \\ & 2h - l_4 - \alpha_2) + \\ & \sin(2f + 2g - 2h + l_4 + \alpha_2) + \sin(2f + 2g - 2h - l_4 - \alpha_2)]] + (\frac{c^2}{16} - \frac{c}{8} + \frac{1}{16}) [\cos(2f + \\ & 2g - 2h + l_4 + \alpha_1) + \cos(2f + 2g - 2h - l_4 - \alpha_1)]\} \end{aligned} \tag{14}$$

Where

$$c = \cos i , s = \sin i , \epsilon = \frac{1}{2} n_g^2 h_x \text{ and } A_g = -\frac{h_x}{h_+} \tag{15}$$

We expand the trigonometric functions of the true anomaly $\frac{r^2}{a^2}$, $\frac{r^2}{a^2}\cos 2f$ and $\frac{r^2}{a^2}\sin 2f$ of Hamiltonian (14) in series of the trigonometric functions of mean anomaly l , up to order three in the eccentricity e (Smart ,1961), therefore the Hamiltonian becomes in the form

$$H = -\frac{\mu^2}{2L^2} + n_9 L_4 + \epsilon L^4 \sum_{N=-5}^{5}\{ \alpha_N^c \cos(N l + l_4) + \alpha_N^s \sin(N l + l_4)\} \tag{16}$$

Where α_N^c and α_N^s are coefficients depending on Delaunay variables (g, h, H, G) and the dimensionless amplitudes of the wave $\frac{h_x}{h_+}$.

3. The Mathematical Algorithm of Solution

Let the considered system of differential equations in terms of canonical elements under the effect of GW are

$$\frac{dL_i}{dt} = -\frac{\partial H}{\partial l_i} \quad ; \quad \frac{dl_i}{dt} = \frac{\partial H}{\partial L_i} \qquad (i = 1, 2, 3,4) \tag{17}$$

Where L_i and l_i are the set of the Delaunay variable in equations (9) and (13). H is the Hamiltonian of GW in (16) .We use Lie transform (Hori,1966; Kamel,1969) to solve our problem. This method is one of the canonical perturbation methods proposed to build successive canonical transformation for Hamiltonian systems depending on a small parameter based on the consideration of Lie series and Lie transform. The system of equation (17) will be solved up to order three in secular terms by performing two successive canonical transformation $(l_i , L_i; \epsilon) \rightarrow (l'_i , L'_i)$ and $(l'_i , L'_i , \epsilon) \rightarrow (l''_i , L''_i)$ analytic in ϵ at $\epsilon = 0$ to eliminate the short and long period terms, respectively from the Hamiltonian, where the primes indicate the transformed variables. The transformed Hamiltonians are in the form

$$H^* (-, l_2, -, L_i; \epsilon) = H_0^* (L_i) + \sum_{n=1} \frac{\epsilon^n}{n!} H_n^* (-, L_i) \tag{18}$$

$$H^{**} (-, L_i; \epsilon) = H_0^{**} (L_i) + \sum_{n=1} \frac{\epsilon^n}{n!} H_n^{**} (-, L_i) \tag{19}$$

With generators W and W^* expandable as

$$W\,(l_i, L_i;\, \epsilon\,) = \textstyle\sum_{n=0} \frac{\epsilon^n}{n!} W_{n+1}\,(l_i, L_i\,) \tag{20}$$

$$W^*\,(-, l_2, l_3, L_i;\, \epsilon\,) = \textstyle\sum_{n=0} \frac{\epsilon^n}{n!} {W^*}_{n+1}\,(-, l_2, l_3, L_i\,) \tag{21}$$

The algorithm for eliminating the short-period terms (i.e. those depending on l_1 and l_4) determining the new Hamiltonian H^* and the generator W is

Zero order: $H_0{}^* = H_0$

First order: $H_1{}^* = H_{1s}$

$$W_1 = \frac{L^{`3}}{\mu^3} \int H_{1p}\; dl^{`}$$

Second order: $H_2{}^* = H_{2s} + \{H_1 + H_1{}^*, W_1\}_s$

$$W_2 = \frac{L^{`3}}{\mu^3} \int (H_{2p} + \{H_1 + H_1{}^*, W_1\}_p)\, dl^{`}$$

Third order: $H_3{}^* = H_{3s} + \{H_2, W_1\}_s + 2\,\{H_2{}^*, W_1\}_s + 2\,\{H_1, W_2\}_s + \{H_1{}^*, W_2\}_s - \{\{H_1{}^*, W_1\}, W_1\}_s$

Where the brackets represent Poisson bracket and subscripts s and p indicates the secular (averaged over l_1' and l_4') and periodic parts in $l^{`}$. The remaining long-period problem is then governed by a Hamiltonian depending only on (L' , G', H' and g).The second algorithm of canonical transformation to remove the long-period terms (i.e. those depending on g') and determining the new Hamiltonian H^{**} and the generating function W^* is given by

Zero order: $H_0{}^{**} = H_0$

First order: $H_1{}^{**} = H_1{}^*$

Second order: $H_2{}^{**} = H_2{}^*$

$$W_1{}^* = -\frac{1}{2}\, \frac{\partial H_1^{-1}}{\partial G} \int {H^*}_{2p}\; dg^{``}$$

Third order: $H_3{}^{**} = H_3{}^* + \{H_2{}^*, W_1{}^*\}_s + 2\,\{H_2{}^{**}, W_1{}^*\}_s + 2\{H_1{}^*, W_2{}^*\}_s - \{\{H_1{}^{**}, W_1{}^*\}, W_1{}^*\}_s$

$$W_2{}^* = -\frac{1}{3}\, \frac{\partial H_1^{-1}}{\partial G} \int (\{H_2{}^*, W_1{}^*\}_p + 2\,\{H_2{}^{**}, W_1{}^*\}_p + 2\,\{H_1{}^*, W_2{}^*\}_p - \{\{H_1{}^{**}, W_1{}^*\}, W_1{}^*\}_p\,)\, dg^{``}$$

Where the subscripts s and p indicates the secular (averaged over $l^{`}{}_2$ and $l^{`}{}_3$) and the periodic parts in $g^{``}$. Finally, the remaining secular problem, being independent of all angle variables. The elements of the short period are obtained from

$$l_i = l'_i + \epsilon\, l'^1{}_i$$

$$L_i = L'_i + \epsilon\, L'^1{}_i \qquad (i = 1,2,3,4)$$

Where $l'^1{}_i = \frac{\partial W_1}{\partial L'_i}$ and $L'^1{}_i = -\frac{\partial W_1}{\partial l'_i}$ and the prime indicate the transformed terms .The inverse transformations are

$$l'_i = l_i + \epsilon\, l^1{}_i$$

$$L'_i = L_i + \epsilon\, L^1{}_i$$

Where $l^1{}_i = -\, l'^1{}_i$ and $L^1{}_i = -\,L'^1{}_i$ and similarly for the elements of the long period. The secular terms will obtained from the substitution of the new transformed Hamiltonian H^* and H^{**}in equations (18) and (19) into the equations (17). These algorithms are actually very simple, but calculating them by hand is laborious, therefore, all computations were carried out by computer program, using the algebraic manipulation language MATHMATICA V10.

4. Solution and Results

We employ the previous algorithms to obtain analytical expressions for the short-period and long-period terms of the orbital elements; the solution is expressed in terms of the Delaunay variables. We obtain the following results

$$\mathrm{H_0}^* = -\frac{\mu^2}{2\mathrm{L}^2} + n_9\,\mathrm{L}_4 \tag{22}$$

$$\mathrm{H_1}^* = 0 \tag{23}$$

$$\mathrm{W}_1 = \mathrm{L}^7 \sum_{N=-5}^{5} \frac{1}{N+\mathrm{L}^3 n_9} \{ \alpha_N^c \sin(N\,l + l_4) - \alpha_N^s \cos(N\,l + l_4) \} \tag{24}$$

$\mathrm{H_2}^*$ is purely secular and long periodic and $\mathrm{W_1}$ is purely periodic and depends on l and l_4 through the combination $\cos(N\,l + l_4)$ and $\sin(N\,l + l_4)$ thus any Poisson bracket with $\mathrm{W_1}$ or $\mathrm{W_2}$ will be purely periodic and will contribute nothing to $\mathrm{H_3}^*$. The leading terms in $\mathrm{H_2}^*$ must depend only the momenta $(\mathrm{L, G, H})$, but it is depend on the long-period terms g and h consequently it is not possible to perform the second transformation , so The long-period terms will be never eliminated from the Hamiltonian because of l_4. The usual technique is to rely on some sort of numerical integration. We analyze numerically the short –period orbital variations of all the Keplerian orbital elements of four different kinds of orbits, using MATHEMATICA V10. The six elements of each orbit are given in table (1), assuming that binary sources for the gravitational waves with frequency $n_g = 60 \times 10^{-6}$ Hz and amplitude $A = 15 \times 10^{-22}$. Such kinds of frequency of gravitational waves are important since they carry information about how galaxies and black holes co-evolved over the history of the Universe (Prince, 2010). We found an important results; (1) the variation in the semi-major and in the eccentricity is of order 10^{-27} , so we can say that there is not variation in the semi-major and in the eccentricity of all the four orbits due to the gravitational wave. The effect of gravitational wave will be on the inclination, the argument of perigee ω, the longitude of node Ω and on the mean anomaly M .Tables (2), (3), (4), and (5) are represented the amount of the variation of the elements in seconds of degree for all the four orbits on five periods from 0 to 4π. The variation is decreases with period increasing as shown in fig. (2) and fig. (3) for the inclination i and for the argument of perigee ω, the short-period variation of Ω is Similar as ω. The short-period variation of M increases with period increasing as in fig (4).

Table 1: The orbital elements for four different orbits in degree

Orbit	**a**	**e**	i	**ω**	**Ω**	Period
1	35 200 km	0.0045	111^0	110^0	25^0	18.25 h
2	12600 km	0.084	39^0	30^0	100^0	4 h
3	9000 km	0.3	50^0	50^0	60^0	2.5 h
4	7600 km	0.02	60^0	60^0	30^0	2 h

Table 2: The variation of the inclination **i** due to GW in second of degree

Period	**Orbit 1**	**Orbit 2**	**Orbit 3**	**Orbit 4**
0	2.48762	0.585391	0.0227931	3.10296
1	2.48678	0.585158	0.0244145	3.10366
2	2.48595	0.58494	0.0259354	3.10446
3	2.48512	0.584712	0.0273728	3.08798
4	2.48428	0.584498	0.0424745	3.10602
5	2.48097	0.584273	0.030044	3.10683

Table 3: The variation of ***ω*** due to GW in second of degree

Period	**Orbit 1**	**Orbit 2**	**Orbit 3**	**Orbit 4**
0	-1.96926×10^{-12}	-1.72814×10^{-14}	-1.57013×10^{-15}	6.99561×10^{-14}
1	-1.99037×10^{-12}	-1.72328×10^{-14}	-1.61453×10^{-15}	7.05254×10^{-14}
2	-2.01147×10^{-12}	-1.71842×10^{-14}	-1.65892×10^{-15}	7.10943×10^{-14}
3	-2.03255×10^{-12}	-1.71354×10^{-14}	-1.70329×10^{-15}	7.16628×10^{-14}
4	-2.05363×10^{-12}	-1.70866×10^{-14}	-1.74766×10^{-15}	7.22307×10^{-14}
5	-2.07469×10^{-12}	-1.70376×10^{-14}	-1.79201×10^{-15}	7.27983×10^{-14}

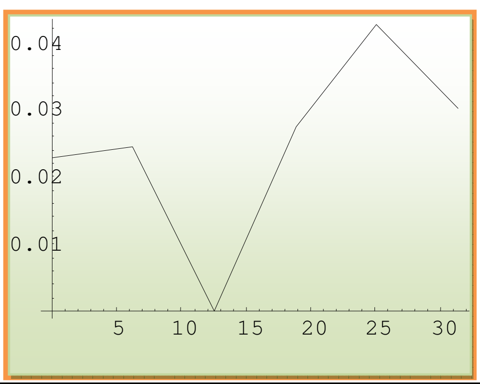
Table 4: The variation of **Ω** due to GW in second of degree

Period	Orbit 1	Orbit 2	Orbit 3	Orbit 4
0	-6.56617×10^{-17}	6.82841×10^{-16}	1.89027×10^{-16}	2.6801×10^{-17}
1	-6.57318×10^{-17}	6.81858×10^{-16}	1.91041×10^{-16}	2.68764×10^{-17}
2	-6.58015×10^{-17}	6.80871×10^{-16}	1.93054×10^{-16}	2.69517×10^{-17}
3	-6.58709×10^{-17}	6.7988×10^{-16}	1.95066×10^{-16}	2.70269×10^{-17}
4	-6.59399×10^{-17}	6.78885×10^{-16}	1.97077×10^{-16}	2.71018×10^{-17}
5	-6.60085×10^{-17}	6.77886×10^{-16}	1.99087×10^{-16}	2.71766×10^{-17}

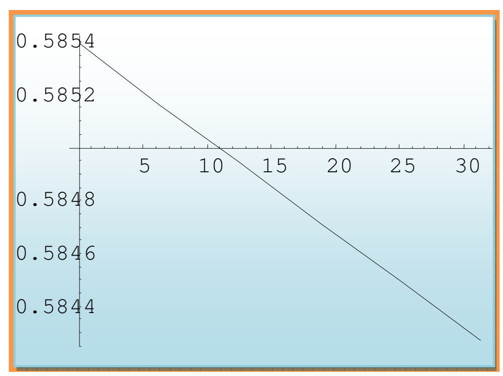
Table 5: The variation of the mean anomaly **M** due to GW in second of degree

Period	Orbit 1	Orbit 2	Orbit 3	Orbit 4
0	1.92029×10^{-15}	1.75276×10^{-14}	1.92029×10^{-15}	-7.00285×10^{-14}
1	1.97825×10^{-15}	1.74781×10^{-14}	1.97825×10^{-15}	-7.05986×10^{-14}
2	2.03619×10^{-15}	1.74285×10^{-14}	2.03619×10^{-15}	-7.11682×10^{-14}
3	2.09411×10^{-15}	1.73788×10^{-14}	2.09411×10^{-15}	-7.17374×10^{-14}
4	2.15203×10^{-15}	1.7329×10^{-14}	2.15203×10^{-15}	-7.23061×10^{-14}
5	2.20993×10^{-15}	1.72791×10^{-14}	2.20993×10^{-15}	-7.28743×10^{-14}

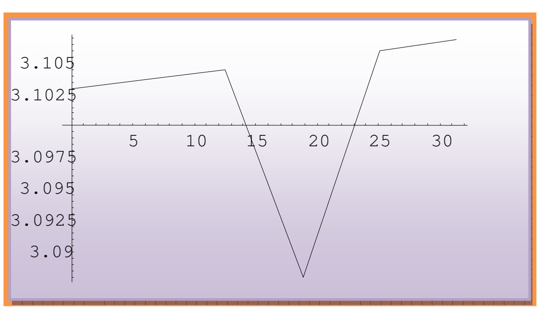
Figure 2: The short period variation of **i** with period for four different orbits where X-axis represents $t - t_0$ in degree and Y-axis represents the variation of each revolution in radiant from 0 to 4π



Orbit 3



Orbit 2



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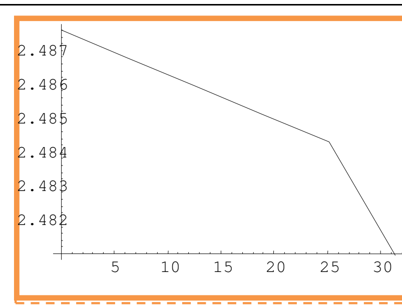


Orbit 1

Figure 3: The short period Variation of $\boldsymbol{\omega}$ for four different orbits where X-axis represents $\boldsymbol{t-t_0}$ in degree and Y-axis represents the variation of each revolution in radiant from 0 to 4π

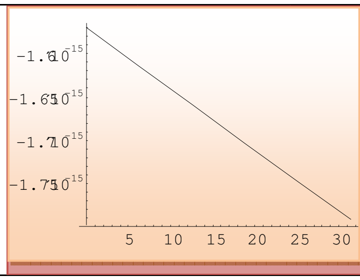
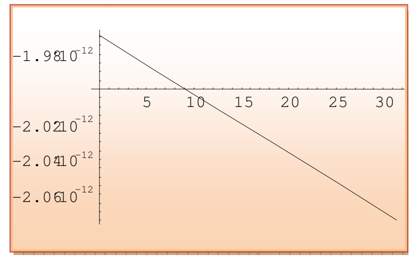
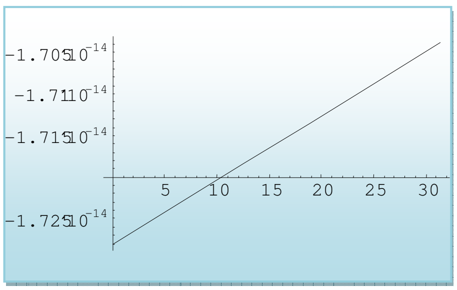
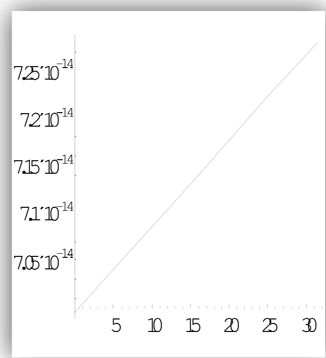
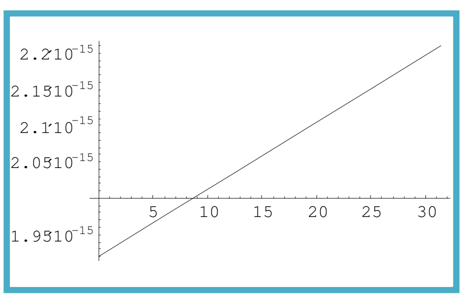
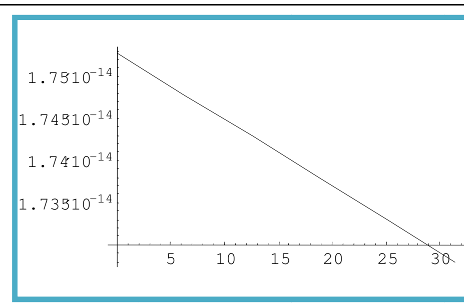
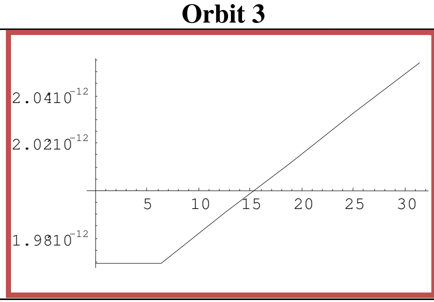
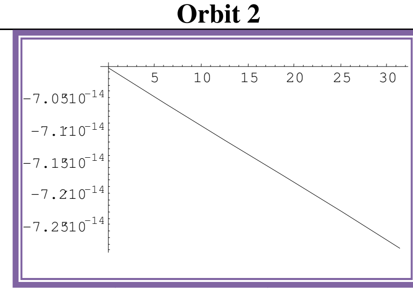
	
Orbit 1	**Orbit 3**
	
Orbit 2	**Orbit 4**

Figure 4: The short period Variation of M for four different orbits where X-axis represents t-t_0 in degree and Y-axis represents the variation of each revolution in radiant from 0 to 4π

	
Orbit 3	**Orbit 2**
	
Orbit 1	**Orbit 4**

5. Conclusion and Discussion

One canonical transformation is used to eliminate the short-period terms. The Short-period variation obtained analytically up to first order in $\in$ and third order in the eccentricity and numerically as in tables (2), (3), (4) and (5). The long-period terms will be never eliminated from the Hamiltonian because of l_4. They can be removed when the following conditions are satisfied, (i) When the effect of the wave is coupled to the wave itself and this occurs only at order six. (ii) Due to commensurability between the wave frequency and the mean motion of the celestial object, when $N\,l = l_4$.**Thus revealing that the effect of gravitational waves is purely of short period. The effect of gravitational wave will be on all the elements of the orbit except the semi-major axis and the eccentricity.** Since the gravitational waves propagated in Z direction, the plane of motion of orbit will affected whatever the shape and the size of the orbit, and consequently there is no effect on the shape and the size of the orbit. The short-period effects decreasing with time within five periods from 0 to 4π except the mean anomaly M variation is increasing as in figures (2),(3),and (4). We have obtained new method and theoretical equations to study the physical effects of gravitational waves from different sources acting on the orbital motion of celestial objects and governing the evolution of the elements.

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