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A Conceptual Problem-Solving Approach for Three-Component Mathematical Mixture Problems – Unraveling the Obscured Method of Alligation Alternate

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Abstract

Amethod based on a conceptual problem-solving approach provided the framework for solving mixture problems. Unlike the method of Alligation Alternate,this method uses concepts and principles to identify the type of variables that are conserved, write equations for the conservation of those variables and use those equations to identify the domain of all variables. Finally, it provided all possible solutions to three-component mixture problems after it wasprogrammed in a Computer Algebra System (CAS).

Keywords: Alligation, Alligation Alternate, mixture problems, mass balance equation, pharmaceutical calculations, physical pharmacy, Conceptual Problem-Solving Approach, pharmacoeconomics, Computer Algebra System, CAS, wxMaxima.

Introduction

In mathematical mixture problems that two types of variables are combined in a single equation, the first type of variable is in the form of amounts, whereas the second variable is used to form arate with respect to the first one. For example, volume could be the first variable and concentration, expressed as the mass over the volume, will be the rate. Mass is the second variable. The distinctiveness of these problems is that there is a preservation of the two variables (volume and mass in our example) before and after combining or mixing them. In other words, the total amounts of both variables remain the same before (initial state) and after the mixing process (final state).Based on this unique feature of the mixture problems two mathematical equations can be constructed that can be utilized to solve for two unknown variables (*eq. 1* and *eq. 2*).

For a *n*-component mixture,

$$V_{a,1} + V_{a,2} + V_{a,3} + \ldots + V_{a,n} = V_{a,f} \tag{1}$$

$$V_{b,1} + V_{b,2} + V_{b,3} + \ldots + V_{b,n} = V_{b,f} \tag{2}$$

Eq. 1 can be modified to include the rate,R, as the ratio $\frac{V_b}{V_a}$.

$$V_{a,1} \cdot \frac{V_{b,1}}{V_{a,1}} + V_{a,2} \cdot \frac{V_{b,2}}{V_{a,2}} + V_{a,3} \cdot \frac{V_{b,3}}{V_{a,3}} + \ldots + V_{a,n} \cdot \frac{V_{b,n}}{V_{a,n}} = V_{a,f} \cdot \frac{V_{b,f}}{V_{a,f}}$$

$$\Rightarrow V_{a,1} \cdot R_1 + V_{a,2} \cdot R_2 + V_{a,3} \cdot R_3 + \ldots + V_{a,n} \cdot R_n = V_{a,f} \cdot R_f \tag{3}$$

In these three equations, V_a and V_b are the types of variables in the initial state, whereas $V_{a,f}$ and $V_{b,f}$ are the variables in the final state of the system.

Typical ratesused in Pharmaceutical Sciences and in Pharmacy Practice could be the concentrations of chemical compounds defined as the ratio of the mass of the solute over the mass or volume of the solvent, molecular dimensions defined as the surface area per molecule, amount of drug released per unit item, defined as the amount of drug released per dosage form or per time and cost per unit item, which could be defined as the amount in dollars per dosage form. The most common type of mixtureproblem presented in pharmaceutical calculations is the one that involves mixing two or more solutions of known concentration to make a certain amount of a third one of the desired concentration. This kind of problems is usually handled by the method of Alligation Alternate.

The method of Alligation Alternate dates back, at least to the beginning of the industrial revolution and it was applied in many scientific areas as well as in the field of banking and economics [1,2]. The main principle of the method involves linking pairs of rates higher and lower than the desired rate to prepare a mixture with an intermediate rate. This is the only conceptual part of the method. The method is empirical and in a number of abstract steps, the user is asked to set the absolute value of the difference between the desired rate from each of the given rates, equal to the amounts of the variables needed to compose a mixture of the desired rate. When it comes to three-component mixtures or higher, other major concerns appear about the effectiveness of the method. The method does not provide all possible answers and itdoes not help the users apply analytical thinking, develop problem-solving strategies using a conceptual approach, build upon the method, expand it and utilize it elsewhere.

In this article, we are approaching two three-component mixture problems as regular mathematical mixture problems. Equations are developed that can be used to represent the different rates and quantities of the variables that are involved in the mixture problems. These equations areused to identify the domain of all variables and develop an algorithm as a step-by-step procedure to provide all possible answers.We also demonstrate how the Alligation Alternate method works and we briefly discuss its limitations.

The Method

Our first example is related to the composition of three-component mixtures. All composition problems of mixtures fall into the area of dilution even if it is a concentration process (one of the solutions or the solvent is more concentrated than the others), and every dilution problem can be managed with a mass preservation equation, even if the original mass of the solute is not preserved during the overall process of dilution. The mass balance or mass preservation equation is commonly used to handle simple dilution problems and it is based on the definition of concentration with respect to the mass and volume of the solute and the solution. This formula can be used to represent the different rates and quantities that are involved in anycomposition-mixture problem, but it is not used to solve three-component mixtureproblems because the number of equations is less than the number of variables. The example below is used to demonstrate the method.

Example 1

Mix 5 %v/v, 45 %v/v and 95 %v/v ethanol solutions to make 1 L of a 65 %v/v ethanol.

Solution

We are given,

$C_1 = 5$ %,
$C_2 = 45$ %,
$C_3 = 95$ %,
$V_f = 1000$ mL,
$C_f = 65$ %

and we are seeking for the volumes of the principal solutions V_1, V_2 and V_3 so that when they are combined they produce 1000 mL of 65 % ethanol solution. Based on this information two equations are constructed:

$$V_1 + V_2 + V_3 = V_f \tag{4}$$

$$V_1 \cdot C_1 + V_2 \cdot C_2 + V_3 \cdot C_3 = V_f \cdot C_f \tag{5}$$

Since we have fewer equations than the number of unknown variables, the problem doesn't have a single solution; instead, it has an infinite number of solutions. It has an *infinite* number of *linearly dependent solutions* because first, the number of variables is more than the number of available equations, and second, the proportions of each rate sum up to a constant value. In linearly dependent three-component systems only two of the variables must be known. One of them has to be assigned a value arbitrarily while the other two are found after solving the system of equations. To avoid having negative values, the solutions domain of all the variables has to be identified. We start with the domain of V_3. Solution 3 is the only one with a concentration higher than the concentration of the desired solution and therefore it will always be included in the mixture. To identify the minimum and maximum values of V_3we set up the values of the other two variables to zero.

Calculation of the minimum volume $V_{3,min}$of C_3.

For $V_{3,min}$ $\Leftrightarrow$ $V_1 = V_{1,min} = 0$

$V_2 = 1000 - V_3 \Rightarrow$ $(1000 - V_3) \cdot 45 + V_3 \cdot 95 = 1000 \cdot 65 \Rightarrow$ $V_3 = V_{3,min} = 400\,\text{mL}$

$\Rightarrow$ $V_2 = V_{2,max} = 600\,\text{mL}$

Calculation the maximum volume $V_{3,max}$of C_3.

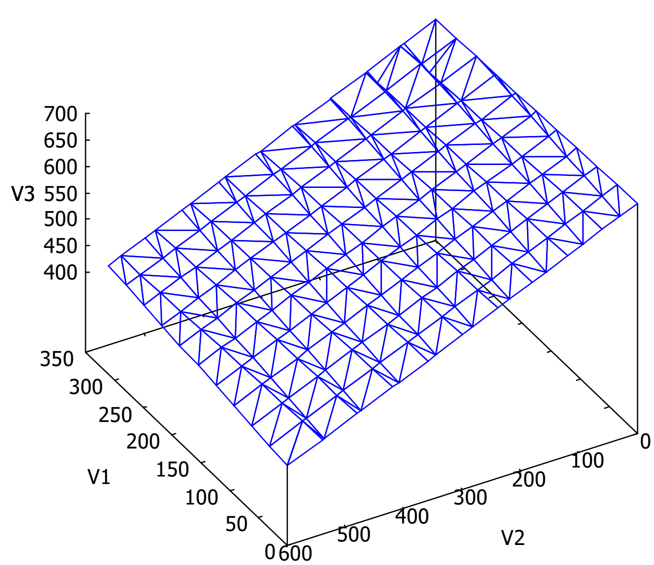
For, $V_{3,max}$ $\Leftrightarrow$ $V_2 = V_{2,min} = 0$

$V_1 = 1000 - V_3 \Rightarrow$ $(1000 - V_3) \cdot 5 + 95 \cdot V_3 = 65000 \Rightarrow$ $V_{3,max} = 666.67\,\text{mL}$

$\Rightarrow$ $V_{1,max} = 333.33\,\text{mL}$

The last two steps have essentially allowed us to identify the domain of *eq. 5* as $\{V_1, V_2, V_3 \mid V_1(0, 333.33),\ V_2(0, 600),\ V_3(400, 666.67)\}$. This is a task that every user should do first. The set of all values that V_1, V_2 and V_3 are allowed to take on so that when they are mixed together they will produce 1 L of 65 % ethanol solution is shown in Figure 1.

Figure 1: The three-dimensional graph of the function $5 \cdot V_1 + 45 \cdot V_2 + 95 \cdot V_3 = 65000$ is plotted with wxMaxima. The volume domains are $0<V_1<333.33$, $0<V_2<600$ and $400<V_3<666.67$



Now that the domain of all variables is known we can proceed to identifying the set of all solutions. This is a labor intensive task that only a CAS can carry out easily and calculate the infinite number of solutions. Manually, you have to vary incrementally the volume of V_3 from 400 mL to 666.67 mL and solve each time *eq. 4* and *eq. 5* to calculate the corresponding values of V_1 and V_2. You could increase the volume by 1 mL or by 0.00001 mL each time; hence the concept of infinite number of solutions. For this example, we have written a do-loop to generate the sequences of V_1, V_2 and V_3 values, using wxMaxima. Results are printed in tabular form for iterations of 1.5 mL V_3 increments (Table 1).

```
C_1:5$
C_2:45$
C_3:95$
C_f:65$
V_f:1000$
V_3:400$
eqn_1: V_1+V_2=1000-(V_3+i*1.5)$
eqn_2: V_1*C_1+V_2*C_2=65000-C_3*(V_3+(i*1.5))$
(print ("V_3 (mL) ","","V_1 (mL)","","V_2 (mL)"),
for i:0 thru 180 do
(S: float(solve([eqn_1,eqn_2],[V_1,V_2])),
print (V_3+(i*1.5),"","","",S))
);
```

The Alligation Alternate Method.

The 95 % is paired with the 45 % and 5 % simultaneously to make the 65 % solution.

95	(65–5)+(65–45) = 80 = V_3
65	95–65 = 30 = V_2
45	95–65 = 30 = V_1
5	140 parts = V_f

$$\frac{140\text{ parts}}{1000\text{ mL}} = \frac{80\text{ parts}}{V_3} = \frac{30\text{ parts}}{V_2} = \frac{30\text{ parts}}{V_1} \Rightarrow V_3 = 571.4\text{ mL},\ V_1 = V_2 = 214.3\text{ mL}$$

Although there are multiple ways to combine the three principal solutions to produce 1 L of the desired one (Table 1), the Alligation Alternate method yields only a single combination which is the special case where $V_1 = V_2$. You may wish to mix separately $V_3$with V_2 and V_3 withV_1 by dividing the 1 L between the two mixtures. That will give you one more combination among the infinite combinations that we have identified using the conceptual problem-solving approach with the mass balance equation. For example, you may mix $V_3$with V_2 to make a 100 mL of desired solution and V_3 withV_1 to make 900 mL of the desired solution. After combining the two mixtures we have 1000 mL of the desired solution (65 %v/v).

95	65 – 45 = 20 parts of V_3
65	95 – 65 = 30 parts of V_2
45	V_f= 50 parts = 100 mL

$$\frac{50\text{ parts}}{100\text{ mL}} = \frac{20\text{ parts}}{V_3} = \frac{30\text{ parts}}{V_2} \Rightarrow V_3 = 40\text{ mL} \qquad V_2 = 60\text{ mL}$$

95	65 – 5 = 60 parts of V_3
65	95 – 65 = 30 parts of V_2
5	V_f= 90 parts = 900 mL

$$\frac{90\text{ parts}}{900\text{ mL}} = \frac{60\text{ parts}}{V_3} = \frac{30\text{ parts}}{V_1} \quad \Rightarrow \quad V_3 = 600\text{ mL} \quad V_1 = 300\text{ mL}$$

The total volumes are V_3 = 640 mL, V_2 = 60 mLV_1 = 300 mL (see Table 1). You will have to use *eq. 5* to verify this combination.

Certainly, the method of Alligation Alternate can produce combinations of the three solutions that upon mixing will produce the desired one, but you cannot plan your experiments. Since the volumes domain has not been determined the maximum volume of each solution that can be combined is not known. The Alligation method allows only the volume of a pair of mixtures to be chosen but not the individual volumes.

The next example is related to pharmacy practice or pharmacoeconomics. It is included here to show that mixture problems are not restricted to pharmaceutics.

Example 2

The pricesfor three multivitamin brands are:
brand 1(R_1): 4.50 cents a tablet
brand 2(R_2): 2.99 cents a tablet
brand 3(R_3): 2.45 cents a tablet
If the cost has to be maintained at 3.5cents per tablet and the available budget (*B*) is \$1050, how much of each brand should be purchased?

Solution

We are given,
R_1 = 4.50 c/tab,
R_2 = 2.99 c/tab,
R_3 = 2.45 c/tab,
R_f = 3.5 c/tab
B = \$1050.00

$$T_f = \frac{B}{R_f} = \frac{105{,}000\text{c}}{3.5\,\frac{\text{c}}{\text{tab}}} = 30{,}000\text{ tab}$$

R_f is the final or average cost rate per tablet. The variables arethe cost and the number of tablets of the three brandsT_1, T_2 and T_3 so that when they are added together they will make 30,000 tablets (T_f) at an average cost rate 3.5 c/tab. Based on this information two equations are constructed:

$$T_1 + T_2 + T_3 = T_f \quad (6)$$

$$R_1 \cdot T_1 + R_2 \cdot T_2 + R_3 \cdot T_3 = R_f \cdot T_f \quad (7)$$

Again, the problem has an infinite number of linearly dependent solutions andthe first task is to identify thedomain of all the variables. R_1 is the only rate with a costhigher than the average cost rate. Therefore, R_1 will always be included in the mixture and the variable T_1 can never be zero. To identify the minimum and maximum values of T_1 we set up the values of the other two variables to zero.

Calculation of the maximum number of tablets,$T_{1,max}$.

For $T_{1,max}$ $\Leftrightarrow$ $T_2 = T_{2,min} = 0$ $\Rightarrow$ $T_1 = T_{1,max} = 15365\text{ tab}$

and $T_3 = T_{3,max} = 14635\text{ tab}$

Calculation of the minimum number of tablets$T_{1,min}$.

For, $T_{1,min}$ $\Leftrightarrow$ $T_3 = T_{3,min} = 0$ $\Rightarrow$ $T_{1,min} = 10133\text{ tab}$

and $T_{2,max} = 19867\text{ tab}$

The domain of this problem is $\{T_1, T_2, T_3 \mid T_1[10133,\ 15365],\ T_2(0,\ 19867],\ T_3(0,\ 14635]\}$. Next, we have to identify the set of all ordered triple solutions within the domain. Similarly to Example 1, we have assigned the value of 10133 to T_1 and written a do-loop in wxMaxima to calculate the values of T_2 and T_3. With every iteration of the loop, T_1 is increased by 60 tablets and T_2 and T_3 are calculated accordingly. All solutions for 60 tablet increments of T_1 are shown in Table 2.

```
R_1:4.5$
R_2:2.99$
R_3:2.45$
R_f:3.5$
T_f:30000$
T_1:10133$
eqn_1: T_2+T_3=30000-(T_1+i*60)$
eqn_2: T_2*R_2+T_3*R_3=105000-R_1*(T_1+(i*60))$
(print ("T_1 (tab) ","","T_2 (tab)","","T_3 (tab)"),
for i:0  thru 90 do
(S: float(solve([eqn_1,eqn_2],[T_2,T_3])),
print (T_1+(i*60),"","","",S))
);
```

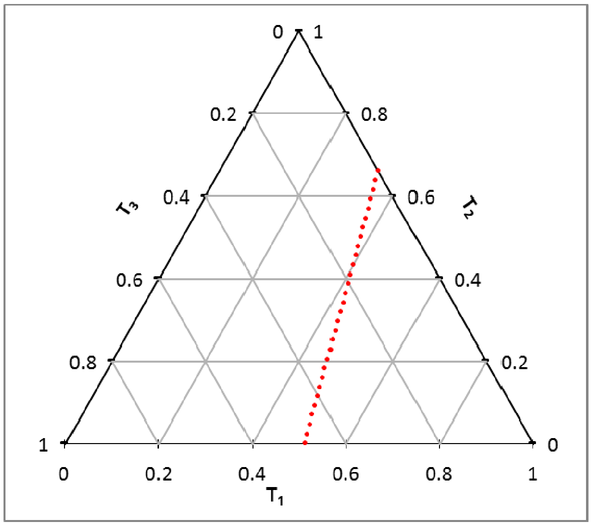
The sets of all solutions shown in Table 2 is a one-to-one correspondence between points in space and ordered triples (T_1, T_2, T_3) in a three-dimensional rectangular coordinate system ($\Re^3$). They are located in the first octant, as the coordinates of all ordered triples are all positive. The solutions can, therefore, be graphed in a triangular plot because the number of Tablets in *eq. 7* cannot be varied independently. Since our system has only two degrees of freedom, it is possible to graph the intersection of all three variables in only two dimensions (Figure 3). Contrary to the conceptual method, the Alligation Alternate method yields only a single triple answer.

The Alligation Alternate Method.

4.5	$0.51+1.05 = 1.56 = T_1$
3.5	$1 = T_2$
2.99	$1 = T_3$
1.45	$3.56 \text{ parts} = T_f$

$$\frac{3.56\text{ parts}}{30000\text{ tablets}} = \frac{1.56\text{ parts}}{T_1} = \frac{1\text{ part}}{T_2 = T_3} \quad \Rightarrow \quad T_1 = 13146\ \text{tab},\ T_2 = T_3 = 8427\text{ tab}$$

Figure 3: Triangular plot of selected ordered triples (divided by 30000) of the three-component system shown in Table 2



Conclusion

The method of Alligation Alternate can be useful to experienced users as a shortcut as it provides quick answers to specific two-component problems but it is of no use to students and the novices. The method does not encourage a strategic analysis of the problem based on principles and it also fails to identify the domain of all the variables in order to provide all possible answers to three- or higher-component mixture problems.It simply provides one single answer after combining higher with lower rates of the principal mixtures. In addition, the multiple empirical rules of the method render it cumbersome and impossible to teach. It is not surprising that students are discouraged and educators are turning awayfrom it. This can only amplify the problem of mathematical deficiencies in pharmacy students.

There is no doubt that mixture problems have a place in research and pharmacy education with specific applications in pharmaceutical sciences andpharmacy practice. Composition mixture problems, problems related to Raoult's law and ideal solutions, and problems in the area of pharmacy practice can be solved with the conceptual problem-solving approach described herein using practical mathematics. For two-component mathematical mixture problems, students that use this method will learn to identify the variables involved in a problem, construct equations related to the preservation of those variables and solve the system of equations to calculate the values of the two variables. For three-component mixture problems or higher, students will learn to construct extended forms of equations to include initial and final states of the system, use these equations to establish the domain of those variables and write an algorithm to calculate all possible solutions to the problem.

Acknowledgments

The algorithms are programmed using the open-source CAS Maxima/wxMaxima.

References

[1] Bregman A. Alligation Alternate and the compositions of medicines: Arithmetic and medicine in the early modern England. Med Hist 2005; 49 (3): 299-320.

[2] Schnaare L R and Stockton J S. Metrology and Pharmaceutical Calculations. In Remington: The Science and Practice of Pharmacy. 22nd ed. Pharmaceutical Press, Philadelphia, PA, 2013.

Table 1: Sets of ordered triple solutions for V_3 increments of 1.5 mL

V_3 (mL)	V_1 (mL)	V_2 (mL)	V_3 (mL)	V_1 (mL)	V_2 (mL)	V_3 (mL)	V_1 (mL)	V_2 (mL)
400	0	600	445	56.25	498.75	490	112.5	397.5
401.5	1.875	596.625	446.5	58.125	495.375	491.5	114.375	394.125
403	3.75	593.25	448	60	492	493	116.25	390.75
404.5	5.625	589.875	449.5	61.875	488.625	494.5	118.125	387.375
406	7.5	586.5	451	63.75	485.25	496	120	384
407.5	9.375	583.125	452.5	65.625	481.875	497.5	121.875	380.625
409	11.25	579.75	454	67.5	478.5	499	123.75	377.25
410.5	13.125	576.375	455.5	69.375	475.125	500.5	125.625	373.875
412	15	573	457	71.25	471.75	502	127.5	370.5
413.5	16.875	569.625	458.5	73.125	468.375	503.5	129.375	367.125
415	18.75	566.25	460	75	465	505	131.25	363.75
416.5	20.625	562.875	461.5	76.875	461.625	506.5	133.125	360.375
418	22.5	559.5	463	78.75	458.25	508	135	357
419.5	24.375	556.125	464.5	80.625	454.875	509.5	136.875	353.625
421	26.25	552.75	466	82.5	451.5	511	138.75	350.25
422.5	28.125	549.375	467.5	84.375	448.125	512.5	140.625	346.875
424	30	546	469	86.25	444.75	514	142.5	343.5
425.5	31.875	542.625	470.5	88.125	441.375	515.5	144.375	340.125
427	33.75	539.25	472	90	438	517	146.25	336.75
428.5	35.625	535.875	473.5	91.875	434.625	518.5	148.125	333.375
430	37.5	532.5	475	93.75	431.25	520	150	330
431.5	39.375	529.125	476.5	95.625	427.875	521.5	151.875	326.625
433	41.25	525.75	478	97.5	424.5	523	153.75	323.25
434.5	43.125	522.375	479.5	99.375	421.125	524.5	155.625	319.875
436	45	519	481	101.25	417.75	526	157.5	316.5
437.5	46.875	515.625	482.5	103.125	414.375	527.5	159.375	313.125
439	48.75	512.25	484	105	411	529	161.25	309.75
440.5	50.625	508.875	485.5	106.875	407.625	530.5	163.125	306.375
442	52.5	505.5	487	108.75	404.25	532	165	303
443.5	54.375	502.125	488.5	110.625	400.875	533.5	166.875	299.625
535	168.75	296.25	580	225	195	625	281.25	93.75
536.5	170.625	292.875	581.5	226.875	191.625	626.5	283.125	90.375
538	172.5	289.5	583	228.75	188.25	628	285	87
539.5	174.375	286.125	584.5	230.625	184.875	629.5	286.875	83.625
541	176.25	282.75	586	232.5	181.5	631	288.75	80.25
542.5	178.125	279.375	587.5	234.375	178.125	632.5	290.625	76.875
544	180	276	589	236.25	174.75	634	292.5	73.5
545.5	181.875	272.625	590.5	238.125	171.375	635.5	294.375	70.125
547	183.75	269.25	592	240	168	637	296.25	66.75
548.5	185.625	265.875	593.5	241.875	164.625	638.5	298.125	63.375
550	187.5	262.5	595	243.75	161.25	640	300	60
551.5	189.375	259.125	596.5	245.625	157.875	641.5	301.875	56.625
553	191.25	255.75	598	247.5	154.5	643	303.75	53.25
554.5	193.125	252.375	599.5	249.375	151.125	644.5	305.625	49.875
556	195	249	601	251.25	147.75	646	307.5	46.5
557.5	196.875	245.625	602.5	253.125	144.375	647.5	309.375	43.125
559	198.75	242.25	604	255	141	649	311.25	39.75
560.5	200.625	238.875	605.5	256.875	137.625	650.5	313.125	36.375
562	202.5	235.5	607	258.75	134.25	652	315	33
563.5	204.375	232.125	608.5	260.625	130.875	653.5	316.875	29.625
565	206.25	228.75	610	262.5	127.5	655	318.75	26.25
566.5	208.125	225.375	611.5	264.375	124.125	656.5	320.625	22.875
568	210	222	613	266.25	120.75	658	322.5	19.5
569.5	211.875	218.625	614.5	268.125	117.375	659.5	324.375	16.125
571	213.75	215.25	616	270	114	661	326.25	12.75
572.5	215.625	211.875	617.5	271.875	110.625	662.5	328.125	9.375

V_3 (mL)	V_1 (mL)	V_2 (mL)	V_3 (mL)	V_1 (mL)	V_2 (mL)	V_3 (mL)	V_1 (mL)	V_2 (mL)
574	217.5	208.5	619	273.75	107.25	664	330	6
575.5	219.375	205.125	620.5	275.625	103.875	665.5	331.875	2.625
577	221.25	201.75	622	277.5	100.5			
578.5	223.125	198.375	623.5	279.375	97.125			

Table 2: Ordered triple solutions for T_1 increments of 60 tablets

T_1	T_2	T_3	T_1	T_2	T_3	T_1	T_2	T_3
10133	19865.46	1.54	11933	13032.07	5034.93	13733	6198.67	10068.33
10193	19637.68	169.31	11993	12804.29	5202.71	13793	5970.89	10236.11
10253	19409.91	337.09	12053	12576.51	5370.49	13853	5743.11	10403.89
10313	19182.13	504.87	12113	12348.73	5538.27	13913	5515.33	10571.67
10373	18954.35	672.65	12173	12120.95	5706.05	13973	5287.55	10739.45
10433	18726.57	840.43	12233	11893.17	5873.83	14033	5059.77	10907.23
10493	18498.79	1008.21	12293	11665.39	6041.61	14093	4831.99	11075.01
10553	18271.01	1175.99	12353	11437.61	6209.39	14153	4604.21	11242.79
10613	18043.23	1343.77	12413	11209.83	6377.17	14213	4376.43	11410.57
10673	17815.45	1511.55	12473	10982.05	6544.95	14273	4148.65	11578.35
10733	17587.67	1679.33	12533	10754.27	6712.73	14333	3920.87	11746.13
10793	17359.89	1847.11	12593	10526.49	6880.51	14393	3693.09	11913.91
10853	17132.11	2014.89	12653	10298.71	7048.29	14453	3465.31	12081.69
10913	16904.33	2182.67	12713	10070.93	7216.07	14513	3237.53	12249.47
10973	16676.55	2350.45	12773	9843.15	7383.85	14573	3009.75	12417.25
11033	16448.77	2518.23	12833	9615.37	7551.63	14633	2781.97	12585.03
11093	16220.99	2686.01	12893	9387.59	7719.41	14693	2554.19	12752.81
11153	15993.21	2853.79	12953	9159.81	7887.19	14753	2326.41	12920.59
11213	15765.43	3021.57	13013	8932.03	8054.97	14813	2098.63	13088.37
11273	15537.65	3189.35	13073	8704.25	8222.75	14873	1870.85	13256.15
11333	15309.87	3357.13	13133	8476.47	8390.53	14933	1643.07	13423.93
11393	15082.09	3524.91	13193	8248.69	8558.31	14993	1415.29	13591.71
11453	14854.31	3692.69	13253	8020.91	8726.09	15053	1187.51	13759.49
11513	14626.53	3860.47	13313	7793.13	8893.87	15113	959.73	13927.27
11573	14398.75	4028.25	13373	7565.35	9061.65	15173	731.95	14095.05
11633	14170.97	4196.03	13433	7337.57	9229.43	15233	504.17	14262.83
11693	13943.19	4363.81	13493	7109.79	9397.21	15293	276.39	14430.61
11753	13715.41	4531.59	13553	6882.01	9564.99	15353	48.61	14598.39
11813	13487.63	4699.37	13613	6654.23	9732.77			
11873	13259.85	4867.15	13673	6426.45	9900.55			